HYDRO RESERVOIR MANAGEMENT FOR AN ELECTRICITY MARKET WITH LONG-TERM CONTRACTS

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To Tracy

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Abstract

This Thesis deals with the management of a mixed hydro-thermal system in a competitive electricity market. A notable feature of our market is the presence of long term financial contracts, or options. We model the energy spot market as a Cournot oligopoly, with a non-competitive fringe. The data from the Cournot model is used in an optimisation model based on Dual Dynamic Programming (DDP). The optimisation model produces operating rules in the form of a marginal water value surface, and these rules guide our medium term simulation model.

We develop a method for using the Cournot model to produce Demand Curves for Release, which describe the amount of water the hydro manager would want to release in a given period for a range of marginal water values (prices). We show how DDP can be thought of as a process of adding demand curves over time, equating marginal costs between periods.

We find that the efficiency of the market is greatly influenced by the size of the contracts, and to a lesser extent by the portfolio of plant that each of the firms has. Increasing contracts lead to increasing output, decreasing spot prices, decreasing profit, increasing consumer surplus, decreasing marginal water values, and increasing storage trajectories. With appropriate choice of contracts the market can be made to mimic perfect competition.

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Chapter 1

Introduction

The New Zealand energy sector has undergone significant reform since the mid 1980s. In 1987 the Electricity Division of the Ministry of Energy became the State Owned Enterprise (SOE) known as the electricity Corporation of New Zealand (ECNZ). At that stage ECNZ generated more than 95 percent of New Zealand's electricity requirements, the remainder being made up by small independent generators, mostly run by local electricity Supply Authorities (ESAs). The transmission system and associated activities were initially run as a subsidiary, Trans Power. In 1994 Trans Power was made an independent SOE. This was seen as an important step in the deregulation of the electricity sector, as the high voltage transmission system is a natural monopoly, and it would be uncompetitive for the generating company to have control over its activities.

In 1996 the reform process went a step further with ECNZ being split into two separate SOEs, ECNZ and Contact, with approximately 60% and 30% of the total generating capacity, respectively. The remaining small generators are being sold off, and plans are now underway for new entry.

On the other side, the ESAs, traditionally owned and run by local body government, have also undergone change. They way has been opened for direct competition, and it is now possible for industrial consumers to purchase electricity from other than their local distributor. What were the ESAs are now separate companies, some still owned by the same local bodies, but others in private hands, and traded on the share market.

In short, the New Zealand electricity sector is now deregulated to such an extent that entry at almost any level is possible. At the outset of this study it was not clear what direction the reforms would take, nor was it clear that they would be as far reaching as they have been. The reform process, and the contribution of OR to it, is discussed by Read (1996).

We set out with the intention of quantifying some of the potential losses in coordination efficiency that deregulation might impose. We decided to concentrate on the reservoir management aspects, as that is an area of particular relevance to New Zealand.

1.1 Background

New Zealand is to some extent following in the footsteps of other nations, especially the U.K. in its moves towards a deregulated electricity sector. This being the case, it would be reasonable to expect that much of the required research into this area would have already been undertaken. This is partly true, but the New Zealand system is somewhat unusual in the world in that up to 75 per cent of our electricity comes from hydro power[1]. We must therefore consider not only the scheduling of thermal stations in correct merit order, but also the reservoir management problem.

Reservoir management for electricity generation[2] is a complicated process. For a start, the value of the energy created using hydro-electric generation is calculated in terms of potential savings from offsets in fuel costs at thermal stations, and of shortage costs. These savings would be made at the marginal fuel cost, and while we may know what the marginal fuel cost is in the current period (it being the cost of generating the last unit of electricity), there will be uncertainty about what the marginal fuel cost will be in future time periods. Because we have the ability to store water from one period to the next, the water may be used to offset fuel costs in future time periods instead of the present one. The value of water

[1] Argentina, Chile and Norway are three other countries with a similar hydro/thermal balance. Of these, Norway is only one with an extensively deregulated electricity sector.

[2] We do not consider other (equally valid) potential uses of water, such as irrigation or recreation in this study.

in any given time period, then, is a function of the marginal fuel costs in this period and all others forward[3]. Not knowing which station will be on the margin, and hence the marginal fuel cost, in future periods thus adds complexity to our problem.

The inflows to the reservoir over future time periods are not known exactly, and must be forecast. This stochastic element of the problem is enough to create many complications in itself.

If, as is often the case, there is more than one hydro reservoir to be considered, then we must consider the operation of these reservoirs jointly. Just as the operation of thermal plant affects the value of water, so does the operation of other hydro plant. In theory each additional reservoir will add a further dimension to the problem. In practice we often chose to model one or two aggregate reservoirs, representing the combined storage of many individual reservoirs. The degree to which this sort of aggregation is valid depends not only on the physical attributes of the system, such as the location of the reservoirs, the constraints on the system, and the correlations that exist among the reservoirs, but also upon the time frame of the model. In a long term planning model it may be reasonable to aggregate many reservoirs together, as long as the system is not expected to be constrained unduly for any long period of time. However, when considering the half-hourly releases from it it is more important that each reservoir be modelled explicitly.

Over the past two decades many computer models have been developed specifically to model various aspects of the New Zealand electricity sector. Aspects to be considered include medium term scheduling, short term scheduling, pricing, and long term planning.

Short term scheduling is where the market co-ordinator must decide how, on a half hourly basis, they will meet the load in a given week, given the information from the medium term schedule. At this level allowances must be made for half hourly load changes, and plant availability constraints. This information would typically only have appeared in an aggregated form in the medium term schedule. In a market situation this will require the Market Co-ordinator (MC) to accept bids for supply and demand, and somehow match these to clear the market.

[3]Actually we need only consider future periods until the reservoir is next completely full or completely empty.

Medium term scheduling is typically done over an annual time horizon divided into weekly intervals. This problem involves deciding how to use the available generation resources and the transmission network to meet the expected load at various load centres, whilst minimising fuel and expected shortage costs.

Long term planning involves decisions such as where and when should new plant be commissioned. Typical long term planning models look between five and thirty years into the future, working with periods of between one month and a year.

The issue of pricing is particularly important in a market environment. Prices are not only a means of transferring wealth, they also give signals and incentives to others. By raising the price of a scarce resource, such as transmission capacity, it is possible to encourage cutting back on the use of that resource, thus reducing problems of congestion. In an electricity system there may be prices for many different resources. Firstly there is the cost of generation, which will usually vary with time of day, and time of year. Then there is the cost of transmission from one point in the network to another. The transmission costs are typically not just a function of the generating and receiving nodes, but also of all other nodes in the transmission network. To add to this complication, AC power has both a real and reactive components, and prices will not only exist for the real and reactive components, but the real prices will interact with the reactive prices. On top of all this, there is the need to provide reserve capacity in case of unexpected plant failure etc. All of these things add up to a very complicated and intertwined pricing problem, which must somehow be dealt with.

The model we consider in this thesis is a medium term one. We concentrate on the reservoir management problem in a market context. We ignore such complexities as transmission losses, transmission constraints, and the pricing and scheduling of spinning reserve, and we model only a single reservoir system. They are ignored not because they would be difficult to model (which may indeed be the case), but because the additional effort of including them in this study would not provide extra insight into the questions we are asking. Refer to (Ring 1996) for a consideration of many aspects of transmission pricing, and to (Drayton-Bright 1997) for a discussion of spinning reserve issues.

The medium term model uses a dual variant of Stochastic Dynamic Programming. Our medium term model (MT) is built over the top of a single period model (SP). SP is a Cournot model of a duopoly. The players in the market bid quantities, and MC clears the market at the market clearing, or spot price. The demand side is modelled via a demand curve.

1.2 Outline of this thesis

This thesis is organised as follows

Chapter 1 This introduction.

Chapter 2 A review of earlier work on modelling electricity markets, and on methods for modelling reservoir management.

Chapter 3 The formulation of our single period model. The model is developed as a Cournot model of an oligopoly. The consumer demand is represented by either a linear or a constant elasticity demand curve.

Chapter 4 Discussion of the sensitivity of the model to various parameters. In particular we note the sensitivity to the elasticity of demand, and to the setting of contracts. We also compare results for linear demand with those for constant elasticity demand.

Chapter 5 Multi-period theory. Here we extend DDP theory to cover general demand and supply curves, and show how DDP is analogous to addition of demand curves over time. We discuss the tractability of this approach to modelling in the New Zealand electricity sector.

Chapter 6 Multi-period results for constant elasticity demand curves. Discussion of impact of contracts, and of market structure.

Chapter 7 Conclusions.

Appendix A Simulation results.

Chapter 2

Modelling Approaches

2.1 Introduction

This study combines two areas of electricity sector modelling. They are the modelling of competitive electricity markets, and the hydro reservoir management problem. In this chapter we consider the previous work in these two areas, and indicate where this study fits into the existing literature.

2.2 Models of Competitive Electricity Markets

Culy, Mayes and Read developed a spreadsheet that models gaming aspects in a spot market. This has been applied both to New Zealand (Culy, Mayes & Read 1990*b*, Culy, Mayes & Read 1990*a*) and to Victoria in Australia (Culy & Read 1994). Their model allows for several competing companies, each acting independently, with perfect information about demand, capacities, contract obligations and variable costs of each other company. The companies each aim for short run profit maximisation, taking no account of future effects of their actions. The strategies they have modelled are pricing up or down by one step, withholding supply in order to push up the marginal cost, or increasing supply offered at the risk of lowering the spot price. Companies may also be specified as price takers, ignoring gaming opportunities and bidding at marginal cost. The spreadsheet models a repeated game in which decisions for the current round are based on the (known) demand for the current round, and on observations of the offers in the previous round. The spreadsheet iterates until an equilibrium is found. The equilibrium is not guaranteed to be unique, and may not be the global optimum. Hydro stations are modelled as thermal stations with an assumed marginal water value.

Klemperer & Meyer (1989) presented a model of an oligopoly facing uncertain demand. The firms' strategies consisted not of fixed prices or quantities as is the case in Bertrand or Cournot models, but instead of a complete supply curve relating quantity to price. They argued that this allows the firm to better adapt to uncertainties, by presenting a range of operating points, instead of just a single point. They showed how in the absence of uncertainty, there exists a multitude of supply function equilibria (SFE), but that with uncertainty the range is greatly reduced, sometimes to a single supply function. Klemperer and Meyer modeled the uncertainty in demand as an exogenous ex post shock which shifts the demand curve horizontally (along the quantity axis). (Green & Newbery (1992) later redefined the uncertainty to be the time axis in the load duration curve, but otherwise closely followed the analysis of Klemperer and Meyer. This allowed them to model load variation throughout the day, which is an important feature of electricity markets.) Klemperer and Meyer proved that the set of SFEs will contain a unique solution if demand can be arbitrarily high with some finite probability (shortage can occur at some cost), and that otherwise it will be a connected set, with known upper and lower bounds. The analysis includes comparative statics, showing that:

> "Firm's equilibrium supply functions are steeper with marginal cost curves that are steeper relative to demand, fewer firms, more highly differentiated products, and demand uncertainty that is relatively greater at higher prices. The steeper are the supply functions firms choose in equilibrium, the more closely competition resembles the Cournot model (which exogenously imposes vertical supply functions — fixed quantities); with flatter equilibrium supply functions, competition is closer to the Bertrand model (which exogenously imposes horizontal supply functions — fixed prices)." (Klemperer & Meyer 1989, page 1243)

The analysis of Klemperer and Meyer seems to be very appropriate to the study of electricity markets, as the generating companies typically have several stations with different marginal costs, and hence a stepped marginal cost curve. However for their solution approach to be tractable one needs to fit simple functional forms to the marginal costs, the market demand and the uncertainty (the LDC). The solution process is also greatly simplified if the firms are identical, as the problem is then reduced from solving a coupled system of differential equations down to solving a single differential equation.

Green and Newbery (Green & Newbery 1992) used the techniques provided by Klemperer and Meyer (Klemperer & Meyer 1989) to analyse the British electricity spot market. In their model, competing firms submitted smooth supply schedules to the spot market, and the market was solved for Nash equilibria as a single shot game. They first developed a theoretical model of a symmetric duopoly, which they extended to include supply constraints. As well as being more realistic, the supply constraints helped narrow down the range of possible SFEs. This model was then adapted to a particular model of an asymmetric duopoly, which is a better representation of the British electricity spot market, in which one the major firm is 50% larger than its rival. Green and Newbery stated that:

> "The level of output was 1.3 percent lower and the price 3.8 percent higher in the asymmetric case than in the symmetric base case, profits were 5 percent higher, and the deadweight losses involved were 30 percent higher." (Green & Newbery 1992, page 941)

As well they noted that the smaller company does much better than its larger rival, the reason being that the larger company has to do much more of the work involved in keeping the price high.

The approach taken by Green and Newbery requires that firms submit a single supply curve covering the entire day. Culy et al. (1990*b*) conducted experiments where the firms were allowed to submit a different offer in each period of the day, and compared this with the requirement that the same supply curve cover the whole day. They found that requiring a single supply curve offer restricted competition and increased the dead-weight loss[1].

Green and Newbery went on to fit their theoretical model to the empirical reality of the British spot market. However, due to the computational complexities

[1]They hypothesize that the reason for this is that it makes firms commit to either low demand or high demand periods, effectively halving the number of competitors in each period.

involved, they limited their simulation to the symmetric case, where competing firms have identical characteristics, stating that:

> "It is an order of magnitude more difficult to solve the pair of equations for the asymmetric equilibrium than to solve the single equation for the symmetric equilibrium ..." (Green & Newbery 1992, page 941).

For this reason, and the requirement that a single supply curve cover the whole day, we have not used the techniques of Klemperer and Meyer in this project. One of the main aims of this study was to consider what the impact would be of varying the allocation of power stations to the companies, and for this we need to consider asymmetric markets.

Green and Newbery's empirical model used a piece-wise quadratic cost function for the firms[2], and a linear demand curve, with a range of slopes (implying different elasticities). The duopoly model was compared with a quintopoly. The results suggested an average price of £27/MWh for the quintopoly, up from £24 for marginal cost pricing. The estimates for the duopoly ranged from £32 to £66, depending on the slope of the demand curve. While these latter results may seem extreme, it should be noted that they do not take account of the threat of entry or the threat of regulation. Green and Newbery went some way to account for these threats by rebasing their model several years into the future, allowing for new entry, and adapting the cost functions accordingly[3]. The new entrants were assumed to price at marginal cost, since they are not likely to be large enough to hold significant market power. This revised model predicted prices of £21.7 for marginal cost pricing, £26.7 for quintopoly, and a range from £29.7 to £30.1 for duopoly, depending upon the slope of the demand curve. While the sensitivity of the results to the slope of the demand curve was greatly reduced by this new entry, there was still a significant deadweight loss resulting from the shift in price/quantity. The total generation under marginal cost pricing was predicted to be 273 TWh, which gives a total spending on energy of 273 TWh $\times$ £21.7/Mwh $=$ £5924 million. The

[2]As Borenstein & Bushnell (1997) note generator's cost functions are better represented by step functions, but no attempt has been made to extend the supply function equilibria approach to such cost functions.

[3]This only accounts for likely new entrant behaviour once in the market. It does not, for example, give indication of the likely effect these threats would have of putting a cap on the spot price.

deadweight loss was reported as ranging from −£54 million[4] for quintopoly to £108 – £412 million for the duopoly cases. In percentage terms these range from -1 percent to +7 percent of the total energy spending.

A drawback of the analysis of Green & Newbery (1992) is that it doesn't explicitly consider the effects of contracts. However Green (1993) does consider the impact of contracts on the short term market, and goes on to model the market for long term contracts.

Green (1993) used a Cournot model of a symmetric duopoly. The duopolists had quadratic cost functions, and hence linear marginal cost. The residual demand the duopolists faced was assumed to be linear. The market for long-term contracts was modelled as equivalent to Bertrand competition. The equilibrium outcome of the model with limited competition in the pool, but fierce competition in the contract market, was full contracting and marginal cost pricing. Green reported that this result is analogous to that of Allaz & Vila (1993). Recognising that full contracting is an unlikely outcome, Green changed to a Cournot model of the contract market (for ease of modelling) and derived results which showed that:

> "... the effect of even the most limited competition in the contract market will be to increase output in the spot market by up to 20%, and reduce the gap between price and marginal cost by up to 40%. ... The presence of an uncompetitive contract market has produced a substantial increase in welfare, and the gains from the more competitive market that actually exists will be even greater." (Green 1993, page 5)

Green went on to consider the effects of risk aversion, which tended to increase the generators' use of the contract market. Lastly, in a multi-period model Green showed that:

> "... full contracting will not necessarily bring pool prices down to marginal costs if the generators expect to sell future contracts at better prices if they keep the pool prices high." (Green 1993, page 12)

Powell (1993) used a Cournot model of contracting in the electricity industry. He first considered the single period analysis, using a Cournot model with a linear

[4]The negative deadweight loss resulted from a change in investment from the status quo. Under the status quo there was no new investment, and hence less efficient plant was sometimes used.

inverse demand function. He then tackled the issue of recontracting using a mean-variance utility approach. Powell's main conclusions were that a high degree of contracting implies output will be higher and price will be lower than otherwise, and that risk-averse RECs[5] will hedge against shifts in the spot price. The degree of hedging will depend upon the behaviour of the generators. If the generators are truly non-cooperative, then according to Powell the competitive result may emerge. If the generators collude, then spot prices will be above marginal cost, and hedging will only be partial. If the generators collude only through the use of futures, then the degree of hedging may be lower still.

Green (1993) and Powell (1993) both present results indicating that the way in which contracts are re-negotiated is important. While we do not consider the re-negotiating of contracts in this study, we do analyse system operation for a wide range of contracts. See Batstone (1997) for further consideration of contract negotiation and risk management in a hydro system.

Andersson & Bergman (1995) consider a similar situation to Green & Newbery (1992), only this time based on the Swedish electricity market. Their analysis uses a Cournot model, chosen for its ability to easily handle non-symmetric firms, a feature desirable for our analysis too. The time frame for their analysis is two to three years, long enough to cover seasonal variations, but not so long as to be concerned with entry or exit. Although the Swedish market has a large component of hydro generation (up to 50%), and the hydro capacity does vary considerably from year to year, Andersson & Bergman chose to model hydro on an average basis, and have no reservoir management rules in their model. Their analysis shows that given the then existing firm structure and the high degree of concentration (two large firms have between them 75% of capacity) deregulation is not a sufficient condition for lower equilibrium prices, the stated goal. Alternatives such as five equal sized firms (c.f. (Green & Newbery 1992)) or an increase in demand side concentration would, they say, reduce significantly the possibility of the firms influencing the market price.

[5]RECs are the Regional Electricity Companies who buy from the pool.

2.3 Reservoir Management Models

While many reservoir management models have been reported in the past, very few have explicitly considered the reservoir management problem in the context of a competitive wholesale electricity market.

Approaches used in the past have included linear programming (LP), non-linear programming (NLP), dynamic programming (DP) and heuristic methods. The LP, NLP and DP methods have been approached from both a primal and a dual perspective, and many have included some of the stochastic elements of the problem. Because of the complexity of the reservoir management problem it is necessary to take steps to reduce the computational effort involved. One way of achieving this is by leaving out certain aspects, such as stochasticity. Another method is aggregation of similar entities, such as modelling several small reservoirs, each with similar characteristics, as one aggregate reservoir. A third method is decomposition, where the system is modelled as a set of disjoint sub-systems with some scheme for combining the results of these sub-systems together and trading them off against each other. As well as these methods, simulation has been used, often for checking the proposals put forward by the former mentioned methods. Having derived some operating rules, via mathematical programming say, these rules can be checked by simulating the operation of the system.

2.3.1 The New Zealand System

There are several key features which characterise the New Zealand electricity system from a modelling perspective. Probably the most important of these is the large proportion of hydro plant. In 1991, 21 845 Gwh out of a total of 29 556 Gwh were produced by hydro plant (ECNZ 1991). Another major feature is the inter-island DC link between the South Island and the North Island. In the past, the limited capacity of this link has effectively separated the two islands from each other, especially from the point of view of aggregating reservoir storage when modelling.

Since expansion in 1991, though, this has been less of a concern[6]. A third important feature is the stochastic nature of the inflows to the hydro reservoirs. Read & Boshier (1989) reported tests which show that, at least for the New Zealand system, the stochastic nature of the inflows is very important. With these features in mind, we present a selective review of reservoir management models appropriate for the New Zealand system.

Reservoir management models designed for the New Zealand system in the past include the Basic Rule Curve approach (Brudenell & Gilbreath 1959), LP/Network Flows (Boshier & Lermit 1977), Non-Linear Decomposition as used in France by EDF (Read 1979), the STAGE model (Boshier, Manning & Read 1983), and the current system known as PRISM, based on Dual Dynamic Programming (Read & George 1990) in the RESOP model (Read 1989, Culy 1990). More recently Scott & Read (1996) described the model which forms the basis of this thesis.

2.3.2 Competitive Reservoir Models

While there are now many deregulated energy markets throughout the world, there are relatively few where hydro generation is a major part of the overall capacity. Norway, Argentina, Chile, and Brazil are a few notable exceptions. While there has been some recent literature on transmission pricing in Chile (Rudnick, Palmer, Cura & Silva 1995), we are not aware of any on reservoir management under deregulation. There are others where there are large hydro plants with relatively little storage capacity, such as in Victoria, Australia. Without storage there is little inter-temporal linkage, and the reservoir management problem as is faced in New Zealand disappears. In this section we briefly review some of the competitive reservoir mangement models that exist.

In 1991 the Norwegian Parliament moved to deregulate their power market. To quote from (Wiedswang 1993):

> "A new Energy Act came into force on 1 January 1991. This act allows a general third party access, TPA, that is to say free access on equal terms for everyone to transmit power through all Norwegian transmission

[6]Ironically reserve requirements often mean that the link cannot be run at full capacity. When at full capacity the possibility that a pole of the link fails may require greater reserve capacity than is available in the North Island.

> networks. Within their geographical area distribution companies have an obligation to connect all consumers, but no exclusive right to supply power. Distribution companies have no duty to meet an increased consumption in their area through own supplies."

In 1992 99.7 percent of the 117 TWh produced in Norway was from hydro plant. There are more than 200 distribution companies (equivalent to our ESAs). Over 95 percent of the capacity is held by the 34 largest power producers.

The Norwegian Research Institute of Electricity Supply (EFI) have developed methods for the optimal scheduling of a hydro-dominated power system (Johannesen & Flatabø 1989, Flatabø, Olaussen, Hornnes, Haugstad, Johannesen & Nyland 1988). The modelling approaches used at EFI vary with the length of the planning horizon. For the short- and medium-term there is quite detailed representation of individual reservoirs, accounting for head effect, and startup and shutdown costs for thermal plant, with deterministic inflow and demand levels. This is based on network linear programming, with an iterative process to cover the (integer) unit commitment problem. In the long-term the models are deterministic out to a certain point in time, and stochastic from there on. The long-term models are based on a variant of SDP, the "water value method". By using a decomposition approach EFI incorporate their single reservoir model into a "Power Pool Model", in which several subsystems are connected via a power pool, where trading occurs at a spot price. There is no global optimisation, but in an iterative process with some operator intervention the model can be used to improve the operation of the total system.

While the EFI models do allow for trade on the spot market, there is no optimisation, as such, of spot trading. For the mid-term model there is an iterative process, with possible operator intervention, in which spot market bids are adapted until some equilibrium is reached. No attempt is made to look beyond the first order reactions of the other companies.

More recently Halseth (1997) has described a model for analysing market power in the Nordic electricity market. It is based on a Cournot model of the major players in the Nordic market. Halseth considers co-operation (collusion) between companies, and finds (not unexpectedly) that collusion is profitable as long as the major company is involved, and the more companies involved, the better. He does not mention any optimisation of the hydro system, nor does he consider the impact of contracts.

Borenstein & Bushnell (1997) have developed a market simulation model of the California electricity market, with the intention of examining potential for market power. They have done this in response to what they see as a major shortcoming of previous market power studies, that being they have all relied on concentration measures, which fail to account for demand and supply dynamics. They chose a Cournot model to

> "... strike a balance between detailed representation of the costs and incentives of competitors and an explicit, functional representation of the strategies of market participants." (Borenstein & Bushnell 1997, page 7)

As part of their justification for the Cournot model, Borenstein & Bushnell point out that the concept of the Bertrand equilibrium is not really appropriate in a market where infinite expansion is not a realistic possibility. They refer to Kreps & Scheinkman (1983) who argued that when firms first choose capacities, then compete on price, as in the Bertrand model, the outcome may be closely approximated by a Cournot model anyway.

Capacity constraints are an important feature of electricity markets, especially in the short term (outage for maintenance etc.) and in the medium term (it takes years and not weeks to commission new power stations).

Borenstein & Bushnell do not attempt to model entry or exit, nor collusion. They do point out that at least some of the entry and exit activities can be accounted for by varying elasticities, which they do analyse. Their model does include some transmission constraints and losses. They model a competitive fringe of price takers[7] as well as Cournot firms. Hydro generation is not optimally scheduled, although some effort is made via the "peak-shaving" method. This uses quantity as an approximation to marginal revenue, leading to sub-optimal behaviour. In spite of the inaccuracies of the peak-shaving approach Borenstein & Bushnell do have one of the few reported models of a competitive market which includes some

[7]The capacity of the price takers is subtracted from the industry demand curve to leave a residual demand that the Cournot players face.

form of reservoir mangement. There is no consideration given to the influence of contracts.

2.4 Conclusions

It seems that none of the competitive reservoir models explicitly consider the impact of contracts, yet in all studies where contracts have been modelled they have been found to have a significant effect. Although the SFE methods of Klemperer and Meyer seem to fit well within the framework of an electricity market, computational difficulties and the required functional forms make these methods inappropriate for our study. Instead we choose (as did Green, and Allaz ands Vila) to use a Cournot game for the single period model, with explicit consideration of contracts. However, as will be detailed in Chapter 3, we explicitly model each step in the marginal cost curves, and we also consider each sub-period of the LDC separately. We do not attempt to produce a single consistent supply function for each period[8].

We have chosen to use dual stochastic dynamic programming (DSDP) for the long term model for the following reasons:

- it has been applied to the New Zealand system before, and is at the heart of the currently used PRISM and SPECTRA models, allowing for comparison with accepted results,
- it does not require that a well defined underlying objective function exist,
- the results from DSDP lend themselves to graphical interpretation (water value surfaces and guidelines especially),
- DSDP has a nice interpretation in terms of demand curves, which adds some insight to the problem,
- it is relatively simple to implement, and (at least for one or two reservoir models) the computational burden is low enough for today's computers to cope with in reasonable time, and

[8]Drayton-Bright (1997) suggests that such a function does not exist in all cases. In particular, in a constrained hydro system the marginal value of water may well change throughout the day, suggesting different levels of output for the same price in different sub-periods.

- there are no problems with convergence, as there sometimes are in the math programming methods.

To keep our model simple we have chosen to model a single aggregate reservoir, and to ignore transmission constraints, reserve capacity requirements, transmission losses, startup and shutdown costs, and reservoir head effects. Plant availability restrictions and plant efficiency characteristics can be modelled, but not in a dynamic sense.

Chapter 3

Single Period Theory

3.1 Introduction

The New Zealand Electricity Market (NZEM) (EMCO 1996) is a self-regulatory and voluntary environment established in 1994 to (among other things) promote cost minimisation and competitive pricing, allow unbiased new entry, deter anti-competitive behaviour and provide certainty as to future market conditions. The market participants are member companies of the NZEM, and they interact under the rules of the NZEM. The NZEM has three classes of participant, Generator, Purchaser and Trader. Generators supply electricity to the grid. Purchasers consume electricity from the grid, and traders participate in the financial and contractual arrangements of the NZEM. Companies may belong to any or all of the classes.

For our purposes we can divide the electricity market into two sections. On the one hand we have what we will refer to as the *dispatch market* and on the other hand we have the *contract market*.

The dispatch market is the clearing mechanism for the half-hourly dispatch. Generators put in *offers* to sell, and Purchasers put in *bids* to buy, and these are matched to find the market clearing dispatch and price.

The contract market is a financial market providing futures type options on the forthcoming electricity prices. In this study we refer to these as *long term contracts* and we consider them to be pre-determined in the sense that they are outside the scope of our optimisation process.

This Chapter is concerned with the dispatch market, and in particular with individual half-hour sub-periods.

The dispatch market nominally runs from 3 am one day to 3 am the next day. By noon of the day before participants are supposed to have put in offers and bids for the day ahead. The offers are stepped marginal cost (supply) functions which correspond to piecewise-linear cost functions (Figure 3.1. Each thermal unit and each hydro station can offer such a supply curve, with at most five distinct marginal costs in it. There is no provision for linking supply curves either between stations, or inter-temporally, except that thermal generators can specify ramp rate[1] constraints[2]. The bids, like the offers, are stepped demand functions, corresponding to piecewise-linear costs. The purchasers are allowed up to ten tranches in each bid.

The bids and offers are cleared against each other by around 2 pm to give a day ahead schedule. All parties are then free to change their bids, and the market is re-cleared every two hours up until four hours before the dispatch time. From this time participants can only change their bids if they have a good reason[3]. Generators are expected to stick with their offers, and purchasers are supposed to notify the market if their expected loads differ from their bids by more than 10 MW at any node in the network. The market clearing prices are only indicative at this stage.

Immediately prior to dispatch time the market is cleared again, but this time based on forecast load rather than bid load. This forecast is done in much the same way as it was before deregulation, and allows the system operators to adjust for any discrepancies they have good reason to expect to occur. This price is still provisional. The market is the cleared in real time and dispatched by the system operators. The provisional prices are now given based on the bids and offers, and the actual dispatch. However it is not until one month later that the prices are finalised. This gives all participants time to review the events that lead to price shocks etc., and allows for adjustments to be made if needed.

[1]Ramp rates are only accounted for in a forward direction.

[2]There is provision for hydro stations on the same river chain to shift load from one to the other provided the same nett input to the specified nodes on the grid is made.

[3]It is not clear if an equipment failure, a sudden cold snap, or an extreme forecast spot price constitutes a good reason.

Integrated with the market for energy is the market for reserve energy. In order to avoid power cuts in the event of equipment failure a certain amount of plant is kept in a state where it can rapidly provide reserve power in the event of a failure. In New Zealand two categories of reserve are provided, based on how quickly plant can respond. The categories are six and sixty second. We do not consider the market for reserve power in this Thesis, but see (Drayton-Bright 1997) for a detailed analysis of this.

There are also arrangements to compensate generators who are asked to operate contrary to their bids for some reason. In particular generators may be asked to provide reactive power in some regions, and side payments are made in these cases. Again we do not consider this issue, but (Ring 1996) does discuss some aspects of this in the context of nodal pricing.

Perfectly competitive generators will bid into the market at exactly their marginal costs. In the absence of long-term contracts a monopolist or oligopolist would bid in above their marginal cost to extract extra profits. The question we are concerned with in this chapter is how should an oligopolist behave given its contract portfolio and marginal costs?

3.2 Playing Games

When a firm has sufficient market power that they can profitably influence the spot market price, it is natural for that firm to take this into account when deciding how much to offer to the market, and at what price. This issue is well discussed in (Tirole 1988, Friedman 1977, Friedman 1984, Friedman 1983) and many others. Non-cooperative game theory is the most applied theory in this area. The NZEM has some interesting features which are relevant from the game theoretical point of view. In particular, there is a small number of generators (ECNZ and Contact together represent well over 90% of the generating capacity), the station capacities are (at this point in time) well known by everyone, and the marginal costs are (again at this point) reasonably well known. In addition there is a long-term contract market which offers hedges against spot price variations for many years into the future[4].

Traditional analysis has been of two broad types. In the Cournot game the players offer quantities to the market. The market is cleared with the market clearing price coming from the (inverse) consumer demand curve. In the Bertrand game the players offer in prices, and the market is cleared to determine the quantities. More recently (Klemperer & Meyer 1989) developed the idea of supply function equilibria where players offer in entire supply functions (see § 2 for a brief review).

Different analyses are appropriate depending on whether the game is a one off, or is to be repeated until an equilibrium is reached. The usual definition is of the Nash equilibrium:

> A set of actions is in Nash equilibrium if, given the actions of its rivals, a firm cannot increase its own profit by choosing an action other than its equilibrium action. (Tirole 1988, p 206)

The process of putting in bids and offers a day ahead, then revising them every two hours certainly suggests that the NZEM could well be modelled as a multistage game. This process is repeated every day. We now move on to describe the way we have modelled the NZEM.

At the outset of this study the NZEM did not exist, and the exact rules of its operation were far from being formulated. Indeed it was not even clear that the electricity sector would be deregulated to the extent that it has been. Considering as our main goal developing a method for modelling and optimising reservoir management in a deregulated environment[5], we decided that for this study at least it would be reasonable to use a relatively simple model of the single period interaction. To this end we formulated the single period model as a single stage Cournot oligopoly, with the demand side represented by a simple demand curve. Given the relatively large number of purchasers compared with the small number of generators it seems reasonable to model the demand side as pure competition.

[4]ECNZ was required by the Government to offer 87% of its dry year capacity on long-term contract at reasonable prices for up to five years out from the establishment of the electricity market on 1 September 1996. Firm capacity is defined as all thermal and geothermal plant, plus mean hydro capacity.

[5]At the time the basic Cournot model applied to the electricity sector was (fairly) unique too.

3.3 Contracts

Each of the generating companies may have financial contracts with consumers to sell them a pre-arranged quantity of electricity, at a pre-arranged *strike price.* These may be traded via the NZEM, or on some other futures type market. These contracts are private, in the sense that they are arranged outside of the *spot market*, and the strike prices are not directly affected by the spot price in any given period[6].

One way of viewing this transaction is to say that the generating company sells all the electricity it generates to the consumer *at the strike price*, and the consumer onsells any surplus to the spot market *at the spot price.* Equivalently the generator and consumer deal with the spot market, with the consumer being compensated by the generating company (or vice versa) for the difference between the spot price and the contract price on the contract quantity. Technically such a contract is known as a *two way option.* See (Brealey & Myers 1984) or any other corporate finance text for a detailed analysis of financial contracts.

As well as the contracts with the consumers, the generating companies may sell each other one way contracts, known as *call options.* Call options are of the form: we will charge you up to the strike price and no more.

> We agree to compensate you in the event that the spot price is higher than the strike price, and we will compensate you the price difference multiplied by the contract quantity.

These would provide back-up for hydro stations in the event of unexpected shortage, and so we commonly refer to these as *back-up contracts.* The opposite to the call options is the *put options.* Put loosely, these are of the form: you will buy from us at no less than the strike price:

> You agree to compensate us in the event that the spot price is lower than the strike price, and you will compensate us the price difference multiplied by the contract quantity.

Note that these contracts substantially alter generator objectives. In the absence of contracts we would expect generators to have incentives to force prices up by

[6]Spot prices will affect future contract prices in that the value of the contracts depends largely on the expected future spot prices, and these are certainly influenced by current spot prices.

restricting supply. But, as we show later, this incentive is greatly reduced if most sales are covered by contracts. Indeed a generating company which has contracted for more than it can economically produce in a given period will be a nett buyer, and will try to drive the market price down. (See Chapter 4 for examples of this.) We might also expect contracts to alter consumer behaviour, but for the purposes of this study we ignore all demand side second order effects.

3.4 Types of Players

We consider two types of generating companies, or *players*, both of which are likely to appear in a wholesale electricity market. The first is the *perfect competitor*, and the second is the *game player.*

3.4.1 Perfect Competitors

The perfect competitor will generate fully from all stations with marginal costs below the spot price, generate as much as the market requires if marginal, and will not generate anything at all from stations with marginal costs higher than the spot price. Thus the perfect competitor will put in offers corresponding exactly to the generating capacities and marginal costs of its stations. This is the base case in the sense that the perfect competitor makes the most efficient use of the generating resources available. It also corresponds to the perfectly coordinated system operated to minimise fuel cost whilst meeting load, the system in place before deregulation.

3.4.2 Game Players

The second type of player is the *game player*, who is prepared to, and capable of adjusting the offers to the market in order to extract higher profits.

As we will soon see, a game player who is *over-contracted*, that is one who has contracts for more electricity than they can generate at a marginal cost at or below the spot price, will try to lower the spot price, since they will have to buy in the difference between the contract quantity and their capacity *at the spot price.*

Similarly, a game player who is *under contracted* will try to increase the spot price, since they will sell any excess electricity to the market at the spot price.

A game player who is over-contracted is a *nett buyer* on the spot market, and one who is under-contracted is a *nett seller* on the spot market.

Game players and perfect competitors are both assumed to have the objective of maximising their overall profit. Game players are aware that their actions will influence the market, but perfect competitors are either unaware of this possibility, or too small to have any influence[7].

3.5 Formulation of the Full Model

Generating companies are faced with the problem of coordinating their use of hydro and thermal stations throughout the time horizon. They must trade off the use of water now with saving it for use in later periods when it may be more valuable. Limited water storage capacity must be taken into account, as must restrictions on maximum (and possibly minimum) generation levels for each station. In practice, the inflow is not known in advance, and must be forecast, with some uncertainty. Thus the problem is actually stochastic in nature, a complication which we must deal with in the multi-period model.

In words, the objective of each generating company could be described as:

> To maximise revenue minus fuel costs over the whole of the planning horizon, subject to meeting the restrictions on generation levels and water storage in each period.

In this Chapter we restrict our analysis to a single period, ignoring any future effects of our actions.

We have chosen not to model any spatial complexities of the system, such as transmission losses, transmission constraints, and the location of demand and supply (generation) nodes, but see (Ring 1996) for a treatment of these. The demand is modelled by the (inverse) consumer demand curve, $p(g)$, which describes the market price, p, for any given total (i.e. the whole market) generation level, g. We

[7]Note that ownership can make a big difference here. A supply authority with a power station and also with considerable obligation to supply to local consumers will likely behave more like a perfect competitor than a monopolist.

will later (in § 3.7 and § 3.8) consider two specific cases of demand curves, linear demand and constant elasticity of demand.

The total generation for the market is comprised of the generation for all the stations of all the firms. We will index the firms by subscript $j = 1 \dots J$ and each firm's stations by a second subscript $i \in \mathcal{I}$. Let:

$$g_j = \sum_{i \in \mathcal{I}} g_{ji} \tag{3.1}$$

be the total generation for firm j, and

$$\mathrm{g} = \begin{pmatrix} g_1 \\ \vdots \\ g_J \end{pmatrix} \tag{3.2}$$

$$\tag{3.3}$$

be the vector of generation levels for each firm. We will commonly refer to the total generation level for the market by the scalar g:

$$g = \sum_{j=1}^{J} g_j \tag{3.4}$$

Each firm will receive the spot price, $p(g)$ for each unit they generate. In total for each firm this will be:

$$\sum_{i \in \mathcal{I}} g_{ji} p(g) = g_j p(g) \tag{3.5}$$

Each station will have upper and lower bounds on its generation as described by:

$$\underline{g_{ji}} \leq g_{ji} \leq \overline{g_{ji}} \forall\, i \in \mathcal{I}. \tag{3.6}$$

In a single period we can treat the hydro stations the same as thermal stations by assuming some marginal value of water in place of the marginal fuel cost[8].

[8]Although each hydro station will have an associated storage reservoir, in the single period storage bounds can be incorporated into the generation bounds (3.6).

	Capacity	Marginal Cost
Station 1	100	1.0
Station 2	150	2.0
Station 3	100	4.0
Station 4	300	6.0
Station 5	100	7.0

Table 3.1: Station characteristics as shown in Figure 3.1.

Thermal generation comes at a cost, $c_{ji}(g_{ji})$, which we assume to be an increasing function of g_{ji}. Combining the cost functions $c_{ji}(g_{ji})$ together in merit order gives us the cost function for the company:

$$c_j(g_j) \equiv \sum_{i \in \mathcal{I}} c_{ji}(g_{ji}) \tag{3.7}$$

If we ignore start-up and shut-down costs and assume the the efficiency curves to be piece-wise linear, then this function $c_{ji}(g_{ji})$ will be a stepped supply curve. An example is shown in Figure 3.1 for the fictitious station capacities and marginal costs of Table 3.1. Note that there are vertical steps in the marginal cost curve, which means that the marginal cost curve is not strictly a function, and that this will lead to complications when solving for the optimal generation. However it will be sufficient for us to know that the marginal cost is bounded both above and below for these vertical sections.

For a given contract quantity, k_j, a strike price of w_j and a particular spot price, $p(g)$, the station owner will have an obligation of:

$$k_j[p(g) - w_j] \tag{3.8}$$

This can be separated into two parts, those being a fixed revenue of $k_j w_j$ and a variable cost of $k_j p(g)$, the latter varying with the spot price. From a profit maximising point of view, the fixed revenue does not matter, and so we will only include the variable part in our objective function to be maximised.

As we stated earlier, we follow the Cournot assumptions, and hence the decision variable for each firm is the quantity to offer. For the most part of this chapter we will proceed as if our firms have unlimited generating capacity, ignoring

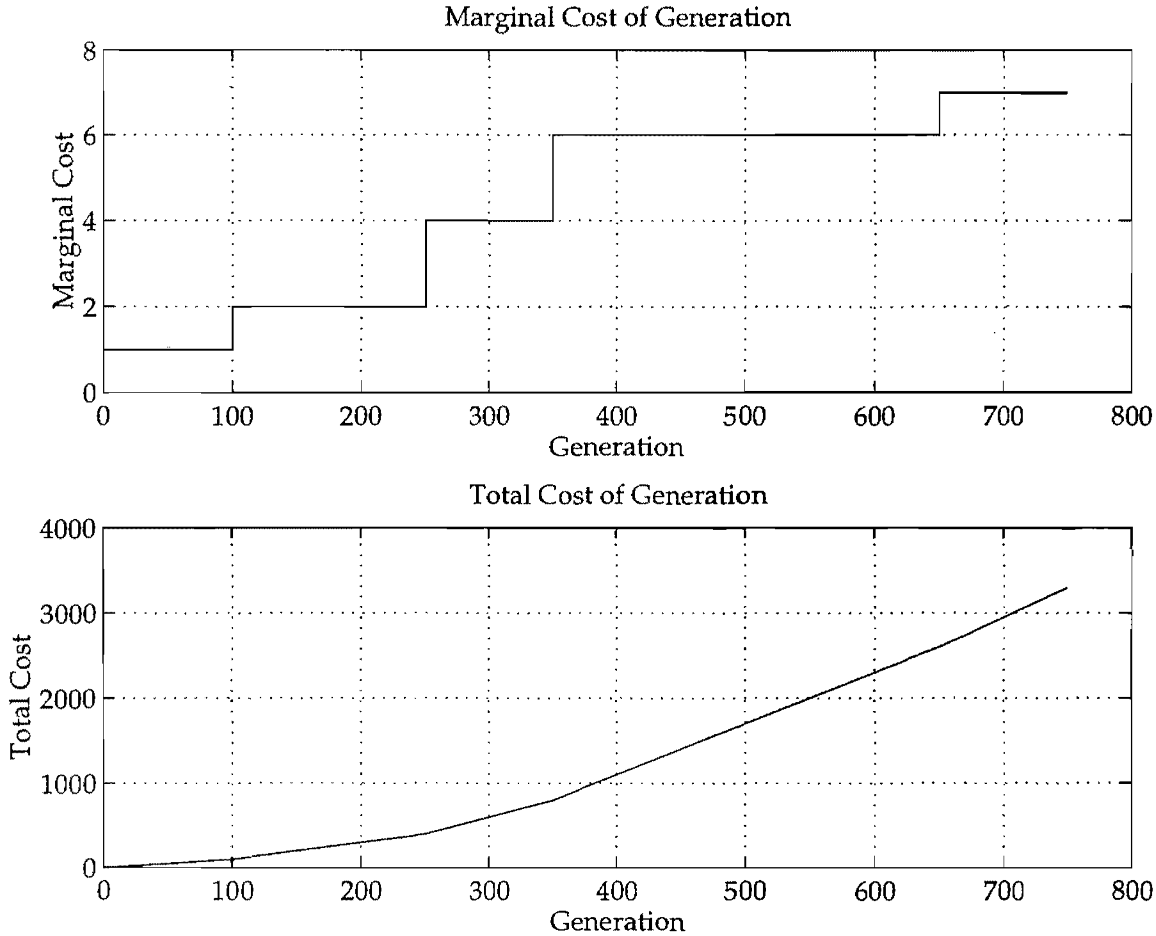

Figure 3.1: A firm's marginal and total generation costs for the stations of Table 3.1.

the generation bounds. We will deal with generation bounds in section § 3.9. Our problem is then:

$$\max_{g_j} \mathcal{L}(g_j) = [p(g)\,[g_j - k_j] - c_j(g_j)] \tag{3.9}$$

Note that this is an unconstrained maximisation problem. At the optimum the first derivative of $\mathcal{L}$ will be zero. Recall the Cournot assumption that each firm will optimise assuming that the other players' outputs are fixed. The first derivative is then:

$$\frac{\partial \mathcal{L}}{\partial g_j} = p(g) + \frac{\partial p}{\partial g_j}[g_j - k_j] - \frac{dc_j}{dg_j} \tag{3.10}$$

Let us define the marginal cost for our company to be

$$\pi_j = \frac{dc_j}{dg_j} \tag{3.11}$$

Setting (3.10) to zero, we get:

$$p(g) = \pi_j - \frac{\partial p}{\partial g_j}[g_j - k_j] \tag{3.12}$$

Hence we have the expected necessary condition that the company should generate at a level which will equate marginal cost with marginal profit.

This result is identical in form to the standard Cournot result except that we have *nett* generation, $g_j - k_j$, instead of g_j. The effect of this is to "distort" output towards the contract quantities. If we are behaving as perfect competitors then we assume that the derivative $\frac{\partial p}{\partial g_j} = 0$, and we simply equate marginal cost with price.

3.6 Market Equilibrium

If we assume that each of the generating companies has an objective function of the form defined above (3.9), and that they each know the others' true marginal fuel costs, then at a market equilibrium equation (3.12) must be satisfied for each of the generating companies simultaneously. The interpretation of this is that each generating company will have reached a local maximum in their profits, that is, a *Nash* equilibrium.

For now let us only concern ourselves with finding equilibria within a given time period. These will later be combined using Dual Dynamic Programming to derive operating strategies for the multi-period problem. If we know the marginal costs and contract levels of each company, and we know explicitly the market demand curve, then we may proceed as follows.

Each player, j, will solve (3.9) to determine their own marginal costs to satisfy (3.12), but at equilibrium each must face the same market price, $p(g)$. Recall that g_j, k_j and π_j are player j's generation, contracts and marginal costs, respectively. Define the market levels as:

$$g \equiv \sum_{j=1}^{J} g_j \tag{3.13}$$

$$k \equiv \sum_{j=1}^{J} k_j \tag{3.14}$$

$$\bar{\pi} \equiv \sum_{j=1}^{J} \frac{\pi_j}{J} \tag{3.15}$$

Now each company must solve equation (3.12):

$$p(g) = \pi_j - \frac{\partial p}{\partial g_j}[g_j - k_j] \qquad \forall\, j = 1, \ldots, J. \tag{3.16}$$

Adding these and dividing through by J yields

$$p(g) = \bar{\pi} - \frac{dp}{dg}\frac{g-k}{J} \tag{3.17}$$

where we have used the fact that in our model we have exactly one (undifferentiated) product, generation, and price is function of the sum of the individual generation levels, so $\frac{\partial p}{\partial g_j} = \frac{dp}{dg} \quad \forall\, j = 1, \ldots, J$. In words, equation (3.17) simply states that the market price will be equal to the average marginal cost plus the change in price due to the average amount of surplus (nett spot) generation.

If we know the marginal costs and contract levels for each player, and we have an expression for market price as a function of total generation, then we may solve equation (3.17) for g, the total generation level. With this we can evaluate $p(g)$, the market price, and substitute this back into equation (3.12) for each player, and hence calculate their individual generation levels, g_j.

Combining and rearranging (3.16) and (3.17) gives us the following market share relationship:

$$\frac{g_j - k_j}{g - k} = \frac{1}{J} + \frac{\pi_j - \bar{\pi}}{\frac{dp}{dg}[g - k]} \tag{3.18}$$

The market share for firm j will be the even share $\frac{1}{J}$ plus an adjustment based on the difference between their marginal cost and the average over all firms, divided by the price differential due to total generation not being exactly equal to total contracts. Note that, as expected, the sum of market shares over all J firms will equal one. It is interesting to consider the effect of the ratio of π_j to $\bar{\pi}$ on the market share. As we have an undifferentiated product, the total nett output, $g - k$ is dependent only on the average levels, $\bar{\pi}$, not on the individual levels, π_j. This means, for example, that two firms with marginal costs of $\pi_1 = 2$ and $\pi_2 = 4$ will produce between them the same total nett output as firms with marginal costs of $\pi_1 = 1$ and $\pi_2 = 5$, but the ratio of $g_1 - k_1$ to $g_2 - k_2$ will be different. Firm one will have a greater market share in the latter case than in the former. Note also the effect of the slope of the demand curve, $\frac{dp}{dg}$ on the ratio of market shares. If the slope were $\frac{dp}{dg} = 2$ in the former example, and $\frac{dp}{dg} = 4$ in the latter, then the market shares would be the same in each case. The two opposing effects are on one hand the deviation of a firm's marginal cost from the average marginal cost, and on the other hand the sensitivty of the consumers to changes in price.

In practice the marginal costs of each company vary with output, and these are usually represented as a stepped curve, such as that in Figure 3.1. This complicates our search for equilibria, as it makes it more difficult to get closed-form solutions to (3.17). However, if we partition the generation for each company into regions of constant marginal cost[9] then we can solve (3.17) in each of these regions,

[9] Also included are the vertical regions of constant output but varying marginal cost.

and look for admissible solutions. By admissible we mean that the generation levels are within the bounds implied by the particular marginal cost. We must then consider the possibility that more than one admissible solution may exist, and this is done in § 3.9.1. For now we will proceed assuming that the marginal cost is not a function of the generation level, and that the firms each have infinite capacity.

In the next section consider market behaviour under the assumption that the market demand can be described by linear demand curves.

3.7 Response Curves Under the Assumption of Linear Demand Curves

Assume that we have a reference point, (p_0, g_0), and that the market demand in any given period can be described by a simple linear relationship between price and demand:

$$p(g) = p_0 + \rho[g - g_0], \tag{3.19}$$

and the slope is then

$$\frac{dp}{dg} = \rho. \tag{3.20}$$

The usual requirement of downward sloping demand implies that $\rho < 0$. Substituting (3.19) and (3.20) into (3.17) gives

$$p_0 + \rho[g - g_0] = \bar{\pi} - \frac{\rho[g - k]}{J}. \tag{3.21}$$

We can rearrange to make g the subject thus

$$\overset{*}{g} = \frac{\bar{\pi}J - p_0 J + \rho g_0 J + \rho k}{\rho[J + 1]}, \tag{3.22}$$

which is the Cournot equilibrium market generation amount. The corresponding price, $\overset{*}{p}$, found by substituting $\overset{*}{g}$ back into (3.19), is

$$\overset{*}{p} = \frac{p_0 + \bar{\pi}J + \rho k - \rho g_0}{J + 1} \tag{3.23}$$

We now have closed form expressions for the market price and generation levels, and with these it is simple to obtain generation levels for individual companies by substituting back into (3.12), giving

$$\begin{aligned}\overset{*}{g}_j &= k_j + \frac{\pi_j - \overset{*}{p}}{\rho} && (3.24)\\ &= k_j + \frac{\pi_j[J+1] - p_0 - \bar{\pi}J - \rho k + \rho g_0}{\rho[J+1]}. && (3.25)\end{aligned}$$

An example might help to clarify the above. Consider the case of two firms $j = \{1, 2\}$ with marginal costs of $\pi_1 = 1.5$ and $\pi_2 = 2.5$, and contract obligations of $k_1 = 800$ and $k_2 = 1200$ with demand curve parameters $p_0 = 3$, $\rho = -10^{-3}$ and $g_0 = 2000$. Hence we have:

$$\begin{aligned}J &= 2 && (3.26)\\ \bar{\pi} &= \frac{1.5 + 2.5}{2}\\ &= 2.0 && (3.27)\\ k &= 800 + 1200\\ &= 2000. && (3.28)\end{aligned}$$

Substituting these values into (3.22) gives us a value for the market generation

$$\overset{*}{g} = \frac{8000}{3}. \quad (3.29)$$

The market price is then (from (3.23))

$$\overset{*}{p} = \frac{7}{3}. \quad (3.30)$$

We can check these for consistency in (3.19)

$$\begin{aligned}\overset{*}{p} &= 3 - 10^{-3} \times \left[\frac{8000}{3} - 2000\right]\\ &= \frac{7}{3}. && (3.31)\end{aligned}$$

Further, the individual generations can now be found using (3.25)

$$g_1 = \frac{4900}{3} \quad (3.32)$$

and

$$g_2 = \frac{3100}{3} \tag{3.33}$$

Note that $g_1 + g_2 = \frac{4900+3100}{3} = \overset{*}{g}$, as expected. This is illustrated in Figure 3.2. Note that both firms are producing, even though the spot price is below π_2. Firm two finds it worthwhile producing at a marginal loss in order to lower the spot price, which they must pay on the shortfall between their generation and contracts. This illustrates the typical scenario. If the spot price is above a firm's marginal production cost then they will produce more than their contracted amount. If the spot price is lower than their marginal cost then they will produce less tha their contracted amount. In general terms, production will fall between the contract amount and the amount that a perfect competitor would produce at the prevailing spot price.

In this section we have derived the Cournot equilibrium conditions for a market with J firms of infinite size, where each firm has some contract commitment, k_j, constant marginal costs, π_j, and the industry demand can be described by a simple linear demand curve. In the next section we repeat this analysis for the case of constant elasticity demand curves.

3.8 Response Curves Under the Assumption of Constant Elasticity of Demand

In this section we consider market behaviour under the assumption that the market demand can be described by constant elasticity demand curves.

Define the price elasticity of demand to be

$$\epsilon \equiv \frac{\partial g}{\partial p}\frac{p}{g} \tag{3.34}$$

where p is the market price, and g is the quantity demanded. Now any point on

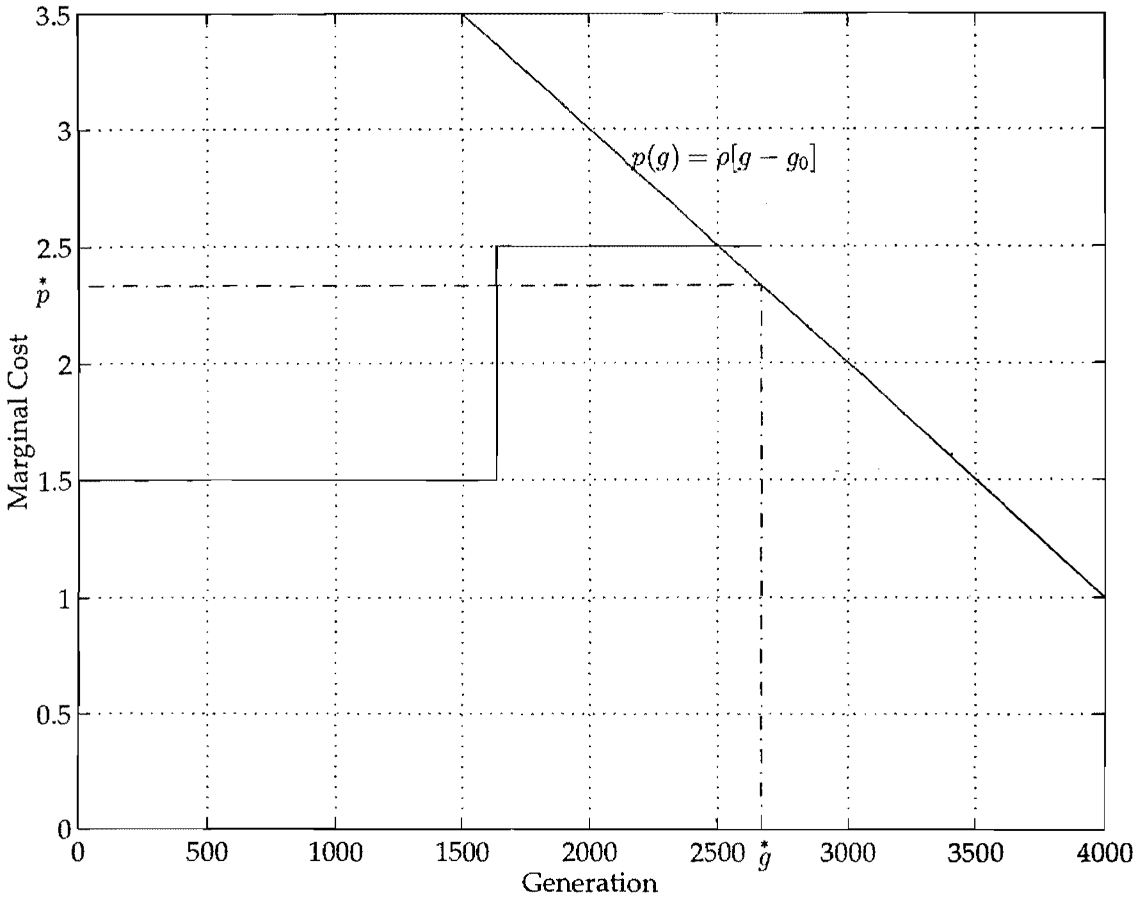

Figure 3.2: Market equilibrium with linear demand.

the demand curve can be described by equation (3.35) or equation (3.36)

$$p(g) = p_0 \left[\frac{g}{g_0}\right]^{\frac{1}{\epsilon}} \tag{3.35}$$

$$g(p) = g_0 \left[\frac{p}{p_0}\right]^{\epsilon} \tag{3.36}$$

where (p_0, g_0) is a reference point somewhere on the demand curve. The slope of the demand curve is simply

$$\frac{dp}{dg} = \frac{p(g)}{\epsilon g}. \tag{3.37}$$

Equations (3.35) and (3.37) give us the description of market price as a function of total generation which we needed in order to solve equation (3.17), although as we shall soon see the result is not quite as simple as in the case of linear demand. Substituting (3.37) into (3.12) we get

$$\begin{aligned}
p &= \bar{\pi} - \frac{p}{\epsilon g}\frac{g-k}{J} \\
\bar{\pi} - p &= \frac{p[g-k]}{\epsilon g J} \\
\epsilon J[\bar{\pi} - p] &= \frac{p[g-k]}{g} \\
\epsilon J[\bar{\pi} - p] - p &= \frac{-pk}{g} \\
g &= \frac{-pk}{\epsilon J[\bar{\pi} - p] - p}.
\end{aligned}$$

We now substitute in (3.36)

$$\begin{aligned}
g_0\left[\frac{p}{p_0}\right]^{\epsilon} &= \frac{pk}{\epsilon J[p - \bar{\pi}] + p} \\
g_0 p^{\epsilon} &= \frac{pkp_0^{\epsilon}}{\epsilon J[p - \bar{\pi}] + p} \\
g_0 p^{\epsilon} &= \frac{kp_0^{\epsilon}}{\epsilon J[1 - \frac{\bar{\pi}}{p}] + 1} \\
p^{\epsilon}[\epsilon J[1 - \frac{\bar{\pi}}{p}] + 1] &= \frac{kp_0^{\epsilon}}{g_0}
\end{aligned}$$

$$[1 + \epsilon J]\overset{*}{p}^{\epsilon} - \epsilon J \bar{\pi} \overset{*}{p}^{\epsilon - 1} - \frac{kp_0^{\epsilon}}{g_0} = 0 \tag{3.38}$$

Equation (3.38) is simply a polynomial in $\overset{*}{p}$, and can be solved easily by a number of numerical methods, although not in closed form. This will tell us the equilibrium market price for the given marginal costs and contract amounts, and from this we can easily determine individual generation levels. A similar polynomial in g could be derived, but since we have (3.38) we might just as well use this value of $\overset{*}{p}$ in (3.36) to get g.

$$\overset{*}{g} = g_0 \left[\frac{\overset{*}{p}}{p_0} \right]^{\epsilon} \tag{3.39}$$

Substituting $\overset{*}{p}$ and $\overset{*}{g}$ into (3.37) we get

$$\left. \frac{dp}{dg} \right|_{\overset{*}{p}, \overset{*}{g}} = \frac{\overset{*}{p}}{\epsilon \overset{*}{g}}. \tag{3.40}$$

We may calculate how much each company will generate using (3.12) to solve for g_j:

$$\begin{aligned} g_j &= \frac{\overset{*}{p} - \pi_j}{\left. \frac{dp}{dg_j} \right|_{p=\overset{*}{p}}} + k_j \\ &= \frac{\overset{*}{p} - \pi_j}{\frac{\overset{*}{p}}{\epsilon \overset{*}{g}}} + k_j \end{aligned} \tag{3.41}$$

As with the linear case, we now present an example. Consider the case of two firms $j = \{1, 2\}$ with marginal costs of $\pi_1 = 1.5$ and $\pi_2 = 2.5$, and contract obligations of $k_1 = 800$ and $k_2 = 1200$ with demand curve parameters $p_0 = 3$, $\epsilon = -\frac{3}{2}$ and $g_0 = 2000$. These values are the same as in the example for the linear demand curve, with ϵ chosen to give the same slope at the reference point (g_0, p_0). Hence we have:

$$J = 2 \tag{3.42}$$

$$\begin{aligned} \bar{\pi} &= \frac{1.5 + 2.5}{2} \\ &= 2.0 \end{aligned} \tag{3.43}$$

$$\begin{aligned} k &= 800 + 1200 \\ &= 2000. \end{aligned} \tag{3.44}$$

Substituting these values into (3.38) gives us a value for the market price

$$\left[1 - \frac{3}{2} \times 2\right] \overset{*}{p}^{-\frac{3}{2}} - \frac{3}{2} \times 2 \times 2 \times \overset{*}{p}^{-\frac{3}{2}-1} - \frac{2000 \times 3^{-\frac{3}{2}}}{2000} = 0 \tag{3.45}$$

which gives

$$\overset{*}{p} = 2.26 \tag{3.46}$$

We can now substitute this value into (3.36) to calculate $\overset{*}{g}$

$$\overset{*}{g} = 3057 \tag{3.47}$$

To calculate the generation for each company, j, we use (3.41):

$$g_1 = 2343 \tag{3.48}$$
$$g_2 = 714 \tag{3.49}$$
$$\tag{3.50}$$

These results are illustrated in Figure 3.3. Note again the features we observed in Figure 3.2, namely that output lies between perfect competition output and contract amount.

In this section we have derived the Cournot equilibrium conditions for a market with J firms of infinite size, where each firm has some contract commitment, k_j, constant marginal costs, π_j, and the industry demand can be described by a constant elasticity demand curve.

3.9 Changing Marginal Costs, and Admissible Solutions.

We now return to the problem of how to proceed when the marginal cost is a function of the generation level. Assume that each firm's marginal cost curve is an arbitrary monotone step curve of the form shown in Figure 3.1. This curve may be partitioned into ranges of generation over which the marginal cost is constant (the horizontal sections of the marginal cost curve) and ranges of marginal cost over which the generation is constant (the vertical steps of the marginal cost curve).

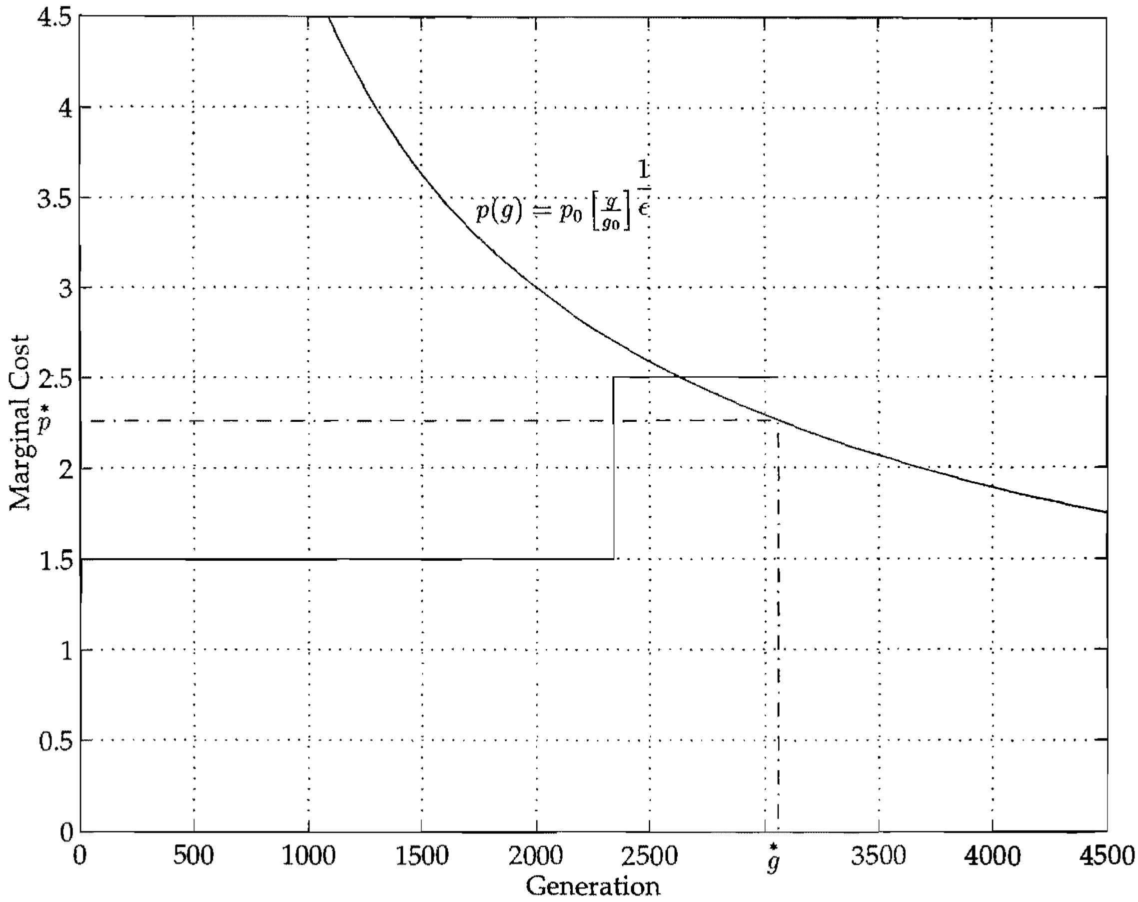

Figure 3.3: Market equilibrium with constant elasticity of demand.

Generation	Marginal Cost
$g = 0$	$0 < \pi < 1.0$
$0 < g < 100$	$\pi = 1.0$
$g = 100$	$1.0 < \pi < 2.0$
$100 < g < 250$	$\pi = 2.0$
$g = 250$	$2.0 < \pi < 4.0$
$250 < g < 350$	$\pi = 4.0$
$g = 350$	$4.0 < \pi < 6.0$
$350 < g < 650$	$\pi = 6.0$
$g = 650$	$6.0 < \pi < 7.0$
$650 < g < 750$	$\pi = 7.0$
$g = 750$	$7.0 < \pi < 50.0$

Table 3.2: Separation of marginal cost function shown in Figure 3.1.

These regions will together they will completely describe the marginal cost curve. Table 3.2 illustrates this for the marginal cost function of Figure 3.1.

We have shown in the previous sections how to find a solution for a fixed marginal cost, and we may use those methods for each of these regions. However the equilibrium generation level for a particular marginal cost may fall outside the range of generation implied by this marginal cost, in which case we say that the solution is not admissible. Our hope is that there will be at least (and preferably exactly) one admissible solution over all the regions.

At this point we will return to the example of § 3.7, except we will now replace the fixed value of π_1 by the range of values described in Table 3.2. All other values will remain the same as before, including π_2.

First as an illustration we present the results for a region where π_1 is a constant. For the second region, where $\pi_1 = 1.0$, the values are:

$$J = 2, \tag{3.51}$$

$$\bar{\pi} = 1.75, \tag{3.52}$$

$$k = 2000, \tag{3.53}$$

$$\overset{*}{g} = \frac{8500}{3}, \tag{3.54}$$

$$\overset{*}{p} = \frac{13}{6}, \tag{3.55}$$

$$g_1 = \frac{5900}{3}, \tag{3.56}$$

and

$$g_2 = \frac{2600}{3}. \tag{3.57}$$

Note that $g_1 = \frac{5900}{3} > 100$, so this solution is inadmissible.

The procedure for the regions where the generation is fixed, but the marginal cost may vary is similar, although not identical. Since the generation for one firm is fixed, regardless of the market price, we may simply subtract this amount from the market demand at all prices. Thus the demand curve is shifted to the left by the amount of fixed generation. Hence we replace g_0 by $g_0 - \Gamma$, where Γ is the total amount of fixed generation (over all firms whose generation is fixed). Equation (3.19) now becomes

$$p(g) = p_0 + \rho[g - g_0 + \Gamma], \tag{3.58}$$

and similarly we have

$$g = \frac{\bar{\pi}J' - p_0 J + \rho J'[g_0 - \Gamma] + \rho k}{\rho[J\prime + 1]}, \tag{3.59}$$

$$\overset{*}{p} = \frac{p_0 + \bar{\pi}J' + \rho k - \rho[g_0 - \Gamma]}{J' + 1}, \tag{3.60}$$

and

$$g_j = k_j + \frac{\pi_j - p_0 - \bar{\pi}J' - \rho k + \rho[g_0 - \Gamma][J' + 1]}{\rho[J\prime + 1]}. \tag{3.61}$$

Note that we have replaced J by J', which we define to be the number of companies whose generation is free to vary. That is, only J' firms are actively competing over this region. Similarly we define $\bar{\pi}'$ and k'.

For the first region, then, where $g = 0$, the calculations are:

$$J' = 1, \tag{3.62}$$

$$\bar{\pi}' = 2.5, \tag{3.63}$$

$$k' = 1200, \tag{3.64}$$

$$\Gamma = 0, \tag{3.65}$$

$$g = 1850, \tag{3.66}$$

$$\overset{*}{p} = 3.15, \tag{3.67}$$

$$g_1 = 0 \tag{3.68}$$

and

$$g_2 = 1850 \tag{3.69}$$

Note that $g = g_2$, as expected. The question we must ask now, is whether or not this is an admissible solution. Does it correspond to a marginal cost $0 < \pi_1 < 1.0$? If we substitute in the values of $\overset{*}{p} = 3.15$, $g_1 = 0$, $k_1 = 800$ and $\frac{dp}{dg} = -10^{-3}$ into (3.12), we have

$$\begin{aligned} \pi_1 &= 3.15 - 10^{-3} \times [0 - 800] \\ &= 3.95 \end{aligned} \tag{3.70}$$

Generation	Marginal Cost	g_1	g_2	π_1	$\overset{*}{p}$	Admissible?
$g = 0$	$0 < \pi_1 < 1.0$	0	1850	3.95	3.15	no
$0 < g < 100$	$\pi_1 = 1.0$	1967	867	1.0	2.167	no
$g = 100$	$1.0 < \pi_1 < 2.0$	100	1800	3.8	3.1	no
$100 < g < 250$	$\pi_1 = 2.0$	1300	1200	2.0	2.5	no
$g = 250$	$2.0 < \pi_1 < 4.0$	250	1725	3.575	3.025	yes
$250 < g < 350$	$\pi_1 = 4.0$	-33	1867	4.0	3.167	no
$g = 350$	$4.0 < \pi_1 < 6.0$	350	1675	3.425	2.975	no
$350 < g < 650$	$\pi_1 = 6.0$	-1367	2533	6.0	3.834	no
$g = 650$	$6.0 < \pi_1 < 7.0$	650	1525	2.975	2.825	no
$650 < g < 750$	$\pi_1 = 7.0$	-2033	2867	7.0	4.167	no
$g = 750$	$7.0 < \pi_1 < 50.0$	750	1475	2.825	4.275	no

Table 3.3: Generation levels and market price for various regions in Firm One's marginal cost function.

which is outside the allowable range of $0 < \pi_1 < 1.0$, so this solution is not admissible.

Table 3.3 summarises these calculations for all the regions. The columns labelled g_1 and π_1 are reproduced in graphical form in Figure 3.4. The plus signs represent the response for a fixed marginal cost (horizontal sections of the supply curve). The asterisks represent the response for a fixed output (vertical sections of supply curve). Observe that the points are co-linear. The slope of the line can be found by differentiating (3.25) to get:

$$\frac{d\overset{*}{g}_j}{d\pi_j} = \frac{J}{\rho\,[J+1]} \tag{3.71}$$

In the case of the example, with $\rho = -10^{-3}$ and $J = 2$, $\frac{d\overset{*}{g}_j}{d\pi_j} = \frac{2000}{3}$. For a point to be an admissible solution it must be on the supply curve for the firm[10]. For a non-decreasing marginal cost curve, and a non-increasing response curve, as we have, it is obvious, graphically at least, that there will be at most one point of intersection[11].

[10]Each of the points was evaluated either with a marginal cost corresponding to a horizontal section of the supply curve, or with an output corresponding to a vertical section. For the point to be admissible it must fall on the corresponding section of the supply curve.

[11]This is not sufficient for a proof of uniqueness of the Nash equilibrium. In terms of § 3.9.1 we have a unique response for a particular value of the other firm's output, giving a single point on the reaction function. That is, the reaction function is a one to one mapping.

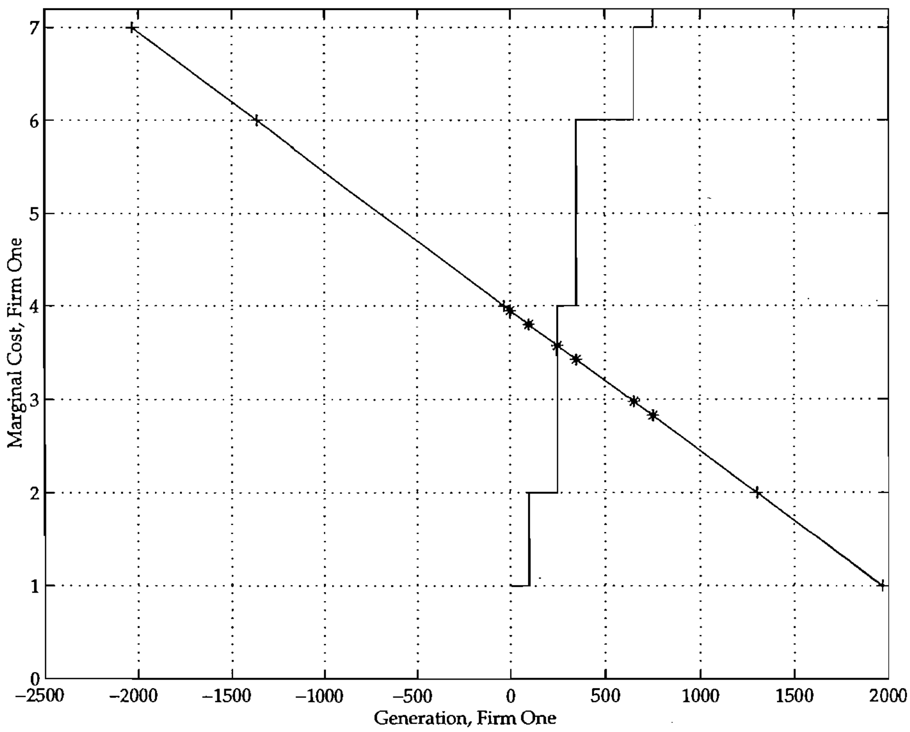

Figure 3.4: Firm One supply curve and generation levels for a range of marginal costs and production levels.

As we had hoped, there is exactly one admissible solution, that being $g_1 = 250$, $g_2 = 1725$, $\pi_1 = 3.575$, $\pi_2 = 2.5$, $g = 1975$, and $\overset{*}{p} = 3.025$. In practice it is not only Firm One which has a marginal cost that varies with generation, so we must construct n-dimensional tables similar to Table 3.3. We must now ask ourselves under what conditions will this there be exactly one admissible solution. This is considered in the next section. Note that the proofs of the following section are based on the idea of reaction functions, and to find the market equilibrium via the methods suggested there would require the construction of separate reaction functions for each step in the marginal cost functions. That would be a prohibitively expensive process, and the methods of this section are a simplification of that process. In our implementation, rather than requiring a complete set of reaction functions we more simply have calculated the intersections of pairs (j-tuplets) of reaction functions. We then examined these for admissible solutions.

3.9.1 Uniqueness of Solution (Linear Demand)

If we assume, as in the previous section, that the marginal cost function, $\pi(g)$, can be split into regions of constant marginal cost, or of constant generation, then there will be a single Cournot equilibrium for the market. Theorems 3.1 and 3.2 formalise this for linear demand. Theorem 3.1 gives us the result that for non-decreasing marginal costs there will be a unique equilibrium. Theorem 3.2 extends this result to cover step-wise marginal cost curves, as we require. Constant elasticity demand is considered in § 3.9.2. The analysis is based on (Tirole 1988), but in our case the profit functions include contracts. The contracts have little effect for the case of linear demand, but do add some complications in the following section where we consider constant elasticity of demand. The proof uses the concept of reaction functions for the firms. A firm's reaction function, R, simply describes its best response to the strategies of the other players. An example is shown in Figure 3.5. It is usual to plot the reaction functions for both players on the same axes, but note that for one of the players the axes are transposed, with the independent variable on the vertical axis.

Theorem 3.1. *Consider the J firm Cournot market with profit functions*

$$z_j(\mathbf{g}) = p(\mathbf{g})[g_j - k_j] - \chi_j(g_j), \tag{3.72}$$

where $p(\mathrm{g})$ is the inverse demand curve defining the market price for given levels of generation, and and $\chi_j(g_j)$ represents the production cost. If

$$p(\mathrm{g}) = p_0 + \rho[-g_0 + \sum_{n=1}^{J} g_n] \tag{3.73}$$

where ρ, p_0 and g_0 are constants, and $\chi_j(g_j)$ is twice continuously differentiable, with $\frac{d\chi_j}{dg_j} > 0$ and $\frac{d^2\chi_j}{dg_j{}^2} \geq 0$, and $\mathrm{g} \geq 0$ then there is a unique Cournot equilibrium vector of generation levels, $\overset{*}{\mathrm{g}}$.

Although a more simple proof is possible by showing that there is only one real valued solution to the first order optimality condition, we use the method suggested in (Tirole 1988). Our reason is that this method will allow us to prove a similar theorem for constant elasticity of demand in the next section, where the former method could not be used.

Proof. The first derivative of $z_j(\mathrm{g})$ with respect to g_j is:

$$\frac{dz_j}{dg_j} = \frac{\partial p}{\partial g_j}[g_j - k_j] + p(\mathrm{g}) - \frac{d\chi_j}{dg_j} \tag{3.74}$$

$$= \rho[g_j - k_j] + p_0 + \rho[-g_0 + \sum_{n=1}^{J} g_n] - \frac{d\chi_j}{dg_j} \tag{3.75}$$

The first order condition that $z_j(\mathrm{g})$ is maximised is that

$$\frac{dz_j}{dg_j} = 0. \tag{3.76}$$

Assume we have some vector, g, of generation levels such that (3.76) is satisfied. That is, Firm j has chosen a generation level g_j as its best reaction to the generation levels of all other firms, $\mathrm{g} \setminus g_j$. Consider now what would happen if Firm i shifted its generation by some small amount, δ_i. This would lead to a change in the LHS of (3.76) of $\delta_i \frac{d^2 z_j}{dg_j dg_i}$. Firm j would be induced to make a small change, $-\delta_j$, to its own generation[12] in order to rebalance 3.76. Thus we would have that

$$\delta_i \frac{d^2 z_j}{dg_j dg_i} = -\delta_j \frac{d^2 z_j}{dg_j{}^2} \tag{3.77}$$

[12] The negative sign indicates that the changes in generation are in opposing directions.

or

$$\frac{\delta_j}{\delta_i} = -\frac{\frac{d^2 z_j}{dg_j dg_i}}{\frac{d^2 z_j}{dg_j{}^2}} \tag{3.78}$$

Equation 3.78 describes the change in g_j for a change in g_i, from a particular point g. It may be rearranged to give a partial derivative $\frac{\partial g_j}{\partial g_i}$, i.e. the slope of Firm j's reaction function. According to (Tirole 1988) a sufficient condition for there to be only one equilibrium is that whenever the reaction curves intercept the absolute value of each of their slopes be less than one. (In Tirole's terminology $|R'| < 1$.) Now

$$\frac{d^2 z_j}{dg_j{}^2} = 2\rho - \frac{d^2 \chi}{dg_j{}^2} \tag{3.79}$$

and

$$\frac{d^2 z_j}{dg_j dg_i} = \rho, \tag{3.80}$$

so

$$R' = -\frac{\frac{d^2 z_j}{dg_j dg_i}}{\frac{d^2 z_j}{dg_j{}^2}} = \frac{\rho}{2\rho - \frac{d^2 \chi}{dg_j{}^2}}, \tag{3.81}$$

and, since by assumption $\frac{d^2 \chi}{dg_j{}^2} \geq 0$, and $\rho < 0$,

$$|R\prime| < 1. \tag{3.82}$$

Since the absolute value of the slope of the reaction function is everywhere less than one we have, by Tirole's argument, precisely one Cournot equilibrium[13].

□

Note that Theorem 3.1 does not rule out the possibility that we have unique equilibria for other marginal cost functions where $\chi_j(g_j)$ is not twice continuously differentiable, as our proof gives sufficient conditions only.

[13]It is conceivable that the reaction functions might not intersect at all, if for example one firm's monopoly output were sufficient that the other firm would not produce anything. In such a case we still have a unique solution, that of the monopoly output. By our requirement that output be non-negative, the reaction functions will always intersect, even in the case just described.

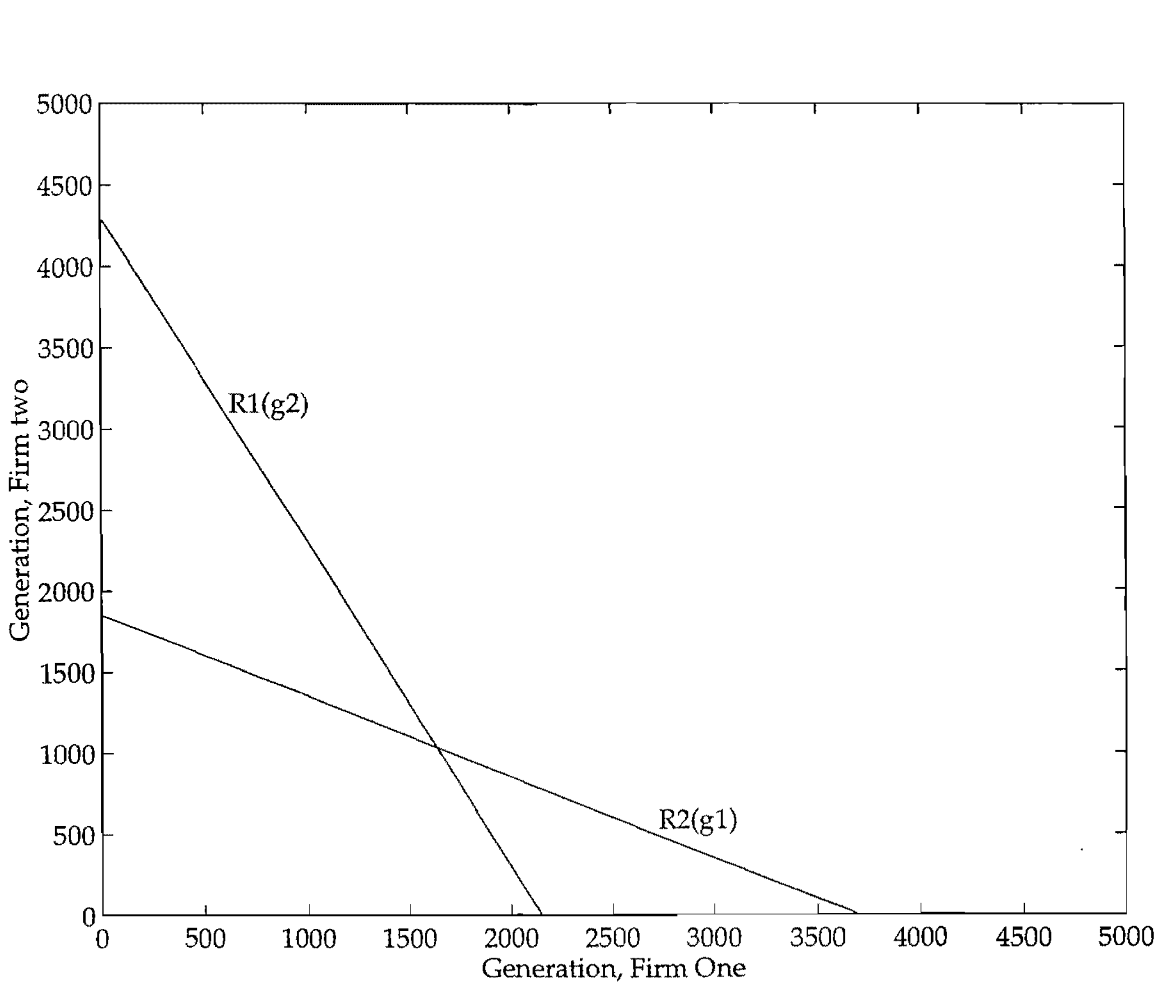

Figure 3.5: Reaction functions for two firms with linear demand.

Figure 3.5 illustrates the reaction functions for the example case of two firms $j = \{1, 2\}$ with marginal costs of $\pi_1 = 1.5$ and $\pi_2 = 2.5$, and contract obligations of $k_1 = 800$ and $k_2 = 1200$ with demand curve parameters $p_0 = 3$, $\rho = -10^{-3}$ and $g_0 = 2000$. As we obtained earlier in § 3.7, the equilibrium point is where $g_1 = \frac{4900}{3}$ and $g_2 = \frac{3100}{3}$.

Theorem 3.1 considers firms whose marginal costs are non-decreasing, and if increasing they increase at an increasing rate. The situation we wish to study is where the marginal cost function can be represented by a step curve, such as Figure 3.1. One approach to modelling this is to construct a separate reaction function for each section of our marginal cost curve, and to switch from one to the next as the stations become fully utilised. Since we always produce less with higher marginal costs than with lower ones, the reaction functions for higher marginal costs will be closer to the origin than those for low marginal costs, and the lines

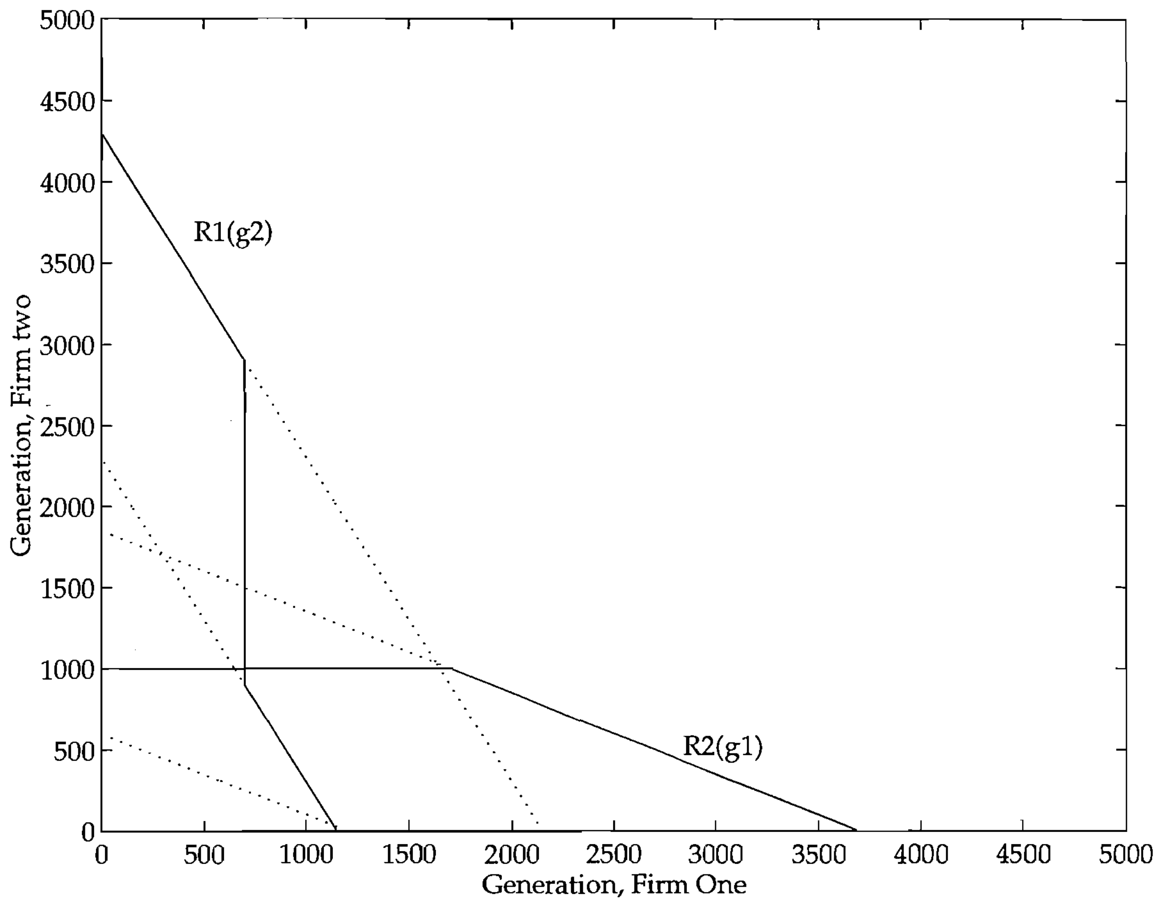

Figure 3.6: Composite reaction functions for two firms with linear demand.

joining the reaction functions will have slope of zero, so Tirole's condition will be satisfied. This is illustrated in Figure 3.6 for the example case of two firms $j = \{1,2\}$ with marginal costs of $\pi_1 = 1.5$ for $g_1 < 700$ and $\pi_1 = 3.5$ for $g_1 > 700$, $\pi_2 = 2.5$ for $g_2 < 1000$ and $\pi_2 = 5$ for $g_2 > 1000$, contract obligations of $k_1 = 800$ and $k_2 = 1200$, and demand curve parameters $p_0 = 3$, $\rho = -10^{-3}$ and $g_0 = 2000$. The equilibrium point is now where $g_1 = 700$ and $g_2 = 1000$. Note that for the higher marginal cost of 5 there is no generation level $g_2 > 1000$, and hence the composite reaction curve for Firm Two is capped at 1000.

Theorem 3.2. *Consider the J firm Cournot market with profit functions*

$$z_j(\mathbf{g}) = p(\mathbf{g})[g_j - k_j] - \chi_j(g_j), \tag{3.83}$$

where $p(\mathbf{g})$ is inverse demand curve defining the market price for given levels of generation, and and $\chi_j(g_j)$ represents the production cost. If

$$p(\mathbf{g}) = p_0 + \rho[-g_0 + \sum_{n=1}^{J} g_n] \tag{3.84}$$

where ρ, p_0 and g_0 are constants, and $\chi_j(g_j)$ is a non-decreasing piece-wise linear function, then there is a unique Cournot equilibrium vector of generation levels, $\mathbf{g}$.

Proof. $\frac{d\chi_j}{d(g_j)}$ is either horizontal (constant marginal cost, increasing production) or vertical (increasing marginal cost, constant production). For the horizontal sections the situation is covered by Theorem 3.1. Each horizontal section corresponds to a different reaction function for Firm j, with higher marginal cost implying lower output. (Note that $\frac{d^2\chi}{dg_j{}^2}$ has a negative coefficient in (3.79).) At the break points in the marginal cost curve we move from one reaction function to the next, keeping output constant. This corresponds to a horizontal line joining the two reaction functions.

The combined reaction function is now a combination of sections which by Theorem 3.1 have $|R'| < 1$, and horizontal sections joining reaction functions, with $|R'| = 0$. Tirole's condition is everywhere satisfied, so there is exactly one equilibrium[14]. □

3.9.2 Uniqueness of Solution (Constant Elasticity Demand)

This section follows § 3.9.1, only now we consider constant elasticity demand curves. We will place the restriction on the demand curves that the elasticity must be between zero and minus one. This is not unrealistic (see § 4.4.1).

Theorem 3.3. *Consider the J firm Cournot market with profit functions*

$$z(\mathbf{g}) = p(\mathbf{g})[g_j - k_j] - \chi_j(g_j), \tag{3.85}$$

[14]Strictly speaking, the slope of the reaction function at the corner points of the step curve is not defined. However we can look at the limiting values approaching the corners from either direction to determine bounds. Since these limits are the two cases of $|R'| < 1$ and $|R'| = 0$ the condition is still satisfied.

where $p(\mathbf{g})$ is inverse demand curve defining the market price for given non-negative levels of generation, and and $\chi_j(g_j)$ represents the production cost. If

$$p(\mathbf{g}) = p_0 \left[\frac{\sum_{n=1}^{J} g_n}{g_0}\right]^{\frac{1}{\epsilon}} \tag{3.86}$$

where ϵ, p_0 and g_0 are constants, $\chi_j(g_j) = \pi_j g_j$ for some positive constant, π_j, and $R_j^{-1}(0) > R_i(0) = g_i^m$ (firm i's output that induces firm j to produce nothing exceeds firm i's monopoly output[15]), then there is a unique Cournot equilibrium vector of positive generation levels, $\mathbf{g}$.

Our proof follows that for Theorem 3.1.

Proof. The first derivative of $z_j(\mathbf{g})$ is:

$$\begin{aligned}\frac{dz_j}{dg_j} &= \frac{\partial p}{\partial g_j}[g_j - k_j] + p(\mathbf{g}) - \frac{d\chi_j}{dg_j} \\ &= \frac{p(\mathbf{g})}{\epsilon \sum_{n=1}^{J} g_n}[g_j - k_j] + p(\mathbf{g}) - \pi \end{aligned} \tag{3.87}$$

As before we can find the slope of the reaction function by implicit differentiation:

$$\frac{d^2 z_j}{dg_j dg_i} = \frac{\partial^2 p}{\partial g_i dg_j}[g_j - k_j] + \frac{\partial p}{\partial g_j} \tag{3.88}$$

$$= \frac{p(\mathbf{g})\left[[g_j - k_j][1-\epsilon] + \epsilon \sum_{n=1}^{J} g_n\right]}{[\epsilon \sum_{n=1}^{J} g_n]^2} \tag{3.89}$$

$$\frac{d^2 z_j}{dg_j{}^2} = \frac{\partial^2 p}{\partial g_j{}^2}[g_j - k_j] + 2\frac{\partial p}{\partial g_j} \tag{3.90}$$

$$= \frac{p(\mathbf{g})\left[[g_j - k_j][1-\epsilon] + 2\epsilon \sum_{n=1}^{J} g_n\right]}{[\epsilon \sum_{n=1}^{J} g_n]^2} \tag{3.91}$$

$$R' = -\frac{\frac{d^2 z_j}{dg_j dg_i}}{\frac{dz_j}{dg_j{}^2}} = -\frac{\frac{p(\mathbf{g})\left[[g_j-k_j][1-\epsilon]+\epsilon\sum_{n=1}^{J} g_n\right]}{[\epsilon\sum_{n=1}^{J} g_n]^2}}{\frac{p(\mathbf{g})\left[[g_j-k_j][1-\epsilon]+2\epsilon\sum_{n=1}^{J} g_n\right]}{[\epsilon\sum_{n=1}^{J} g_n]^2}} \tag{3.92}$$

$$= -\frac{[1-\epsilon]\,[g_j - k_j] + \epsilon \sum_{n=1}^{J} g_n}{[1-\epsilon]\,[g_j - k_j] + 2\epsilon \sum_{n=1}^{J} g_n} \tag{3.93}$$

[15]*Actually our requirement of non-negative output is sufficient for this. In that case we will have some output level of firm i, greater than or equal to their monopoly output that will induce firm j to produce nothing.*

If the numerator of (3.93) is negative then $-1 < R' < 0$, and Tirole's condition is satisfied. However for $g_j > [1-\epsilon]\,k_j - \epsilon\sum_{n\neq j} g_n$ this does not hold, and R' may become positive, and greater than one. However, for this case the reaction functions are always concave in the other player's output (recall that $R' = \frac{dg_j}{dg_i}$):

$$\frac{d^2 g_j}{dg_i{}^2} = \frac{d - \frac{[1-\epsilon][g_j-k_j]+\epsilon\sum_{n=1}^{J} g_n}{[1-\epsilon][g_j-k_j]+2\epsilon\sum_{n=1}^{J} g_n}}{dg_i} \tag{3.94}$$

$$= \frac{-\epsilon\left[[1-\epsilon]\,[g_j-k_j]+2\epsilon\sum_{n=1}^{J} g_n\right]+2\epsilon\left[[1-\epsilon]\,[g_j-k_j]+\epsilon\sum_{n=1}^{J} g_n\right]}{\left[[1-\epsilon]\,[g_j-k_j]+2\epsilon\sum_{n=1}^{J} g_n\right]^2} \tag{3.95}$$

$$= \frac{\epsilon\,[1-\epsilon]\,[g_j-k_j]}{\left[[1-\epsilon]\,[g_j-k_j]+2\epsilon\sum_{n=1}^{J} g_n\right]^2}. \tag{3.96}$$

The denominator of (3.96) is a square, and so is positive. The elasticity of demand, ϵ, is negative, and the generation and contracts are non-negative, so as long as $g_j > k_j$, the numerator is negative. Since we are only concerned with the region where $g_j > [1-\epsilon]\,k_j - \epsilon\sum_{n\neq j} g_n$, we certainly have that $g_j > k_j$, and the numerator, and hence the whole expression is negative. This means that the slope of R' is always decreasing, and hence R' is concave[16] (for the region where its slope may be positive).

We already know that $R(0) \geq 0$ (monopoly output is non-negative) and $R(x) = 0$ for some sufficiently large x (if the market is saturated we will produce nothing). Thus the reaction function intercepts with both axes. We also have that the slope of R' is always decreasing, but it is never less than minus one.

For two concave reaction functions to intercept with each other more than once we need either that at least one of the reaction functions does not intercept with both (positive) axes, or that at least one has a slope more negative than minus one. These two situations are illustrated in Figure 3.7 and 3.8, respectively. Neither of these situations apply, hence we have no more than one equilibrium point. Also, since by assumption $R_j{}^{-1}(0) > R_i(0)$ the curves must intercept at least once, and we must have exactly one equilibrium point. □

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[16] What this means is that the curve cannot turn back on itself to intersect with the other reaction function again. See Figure 3.9.

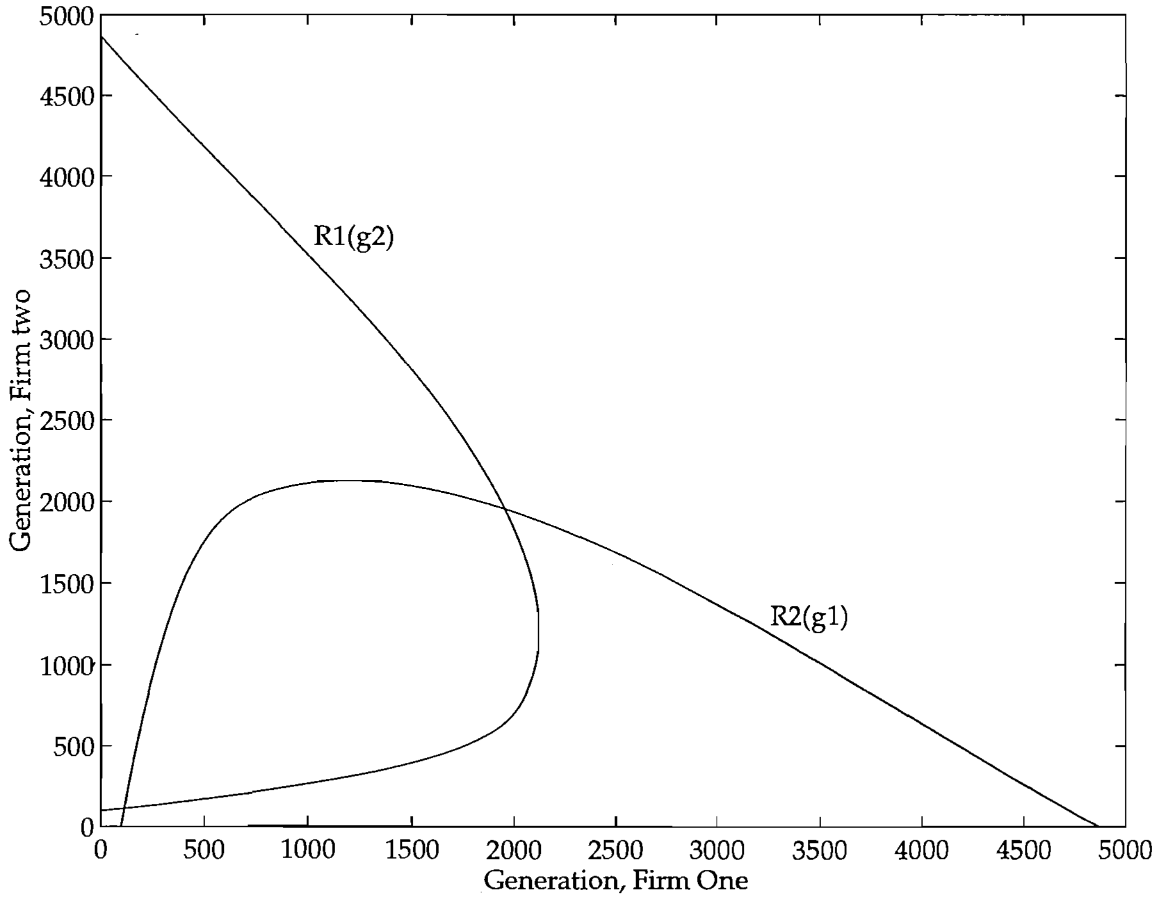

Figure 3.7: Reaction functions not touching both positive axes.

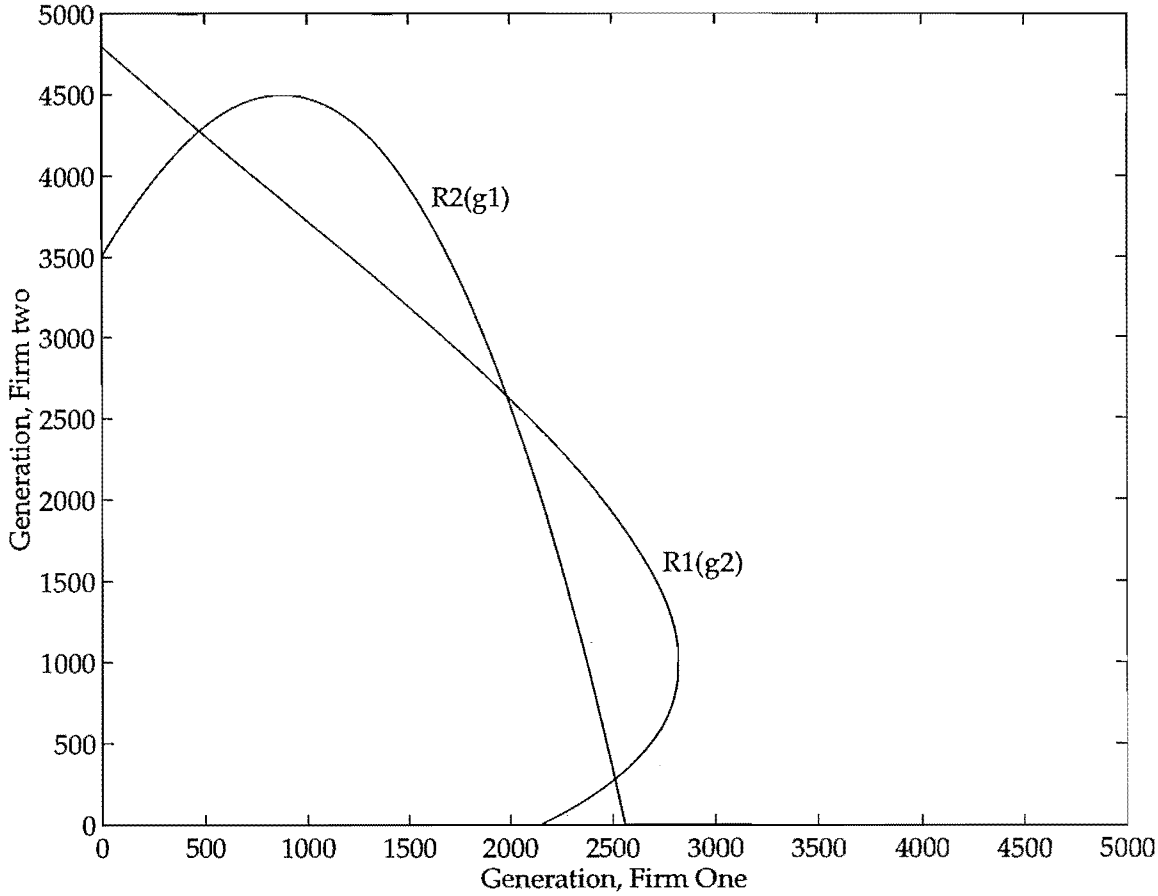

Figure 3.8: One reaction function with $R' < -1$

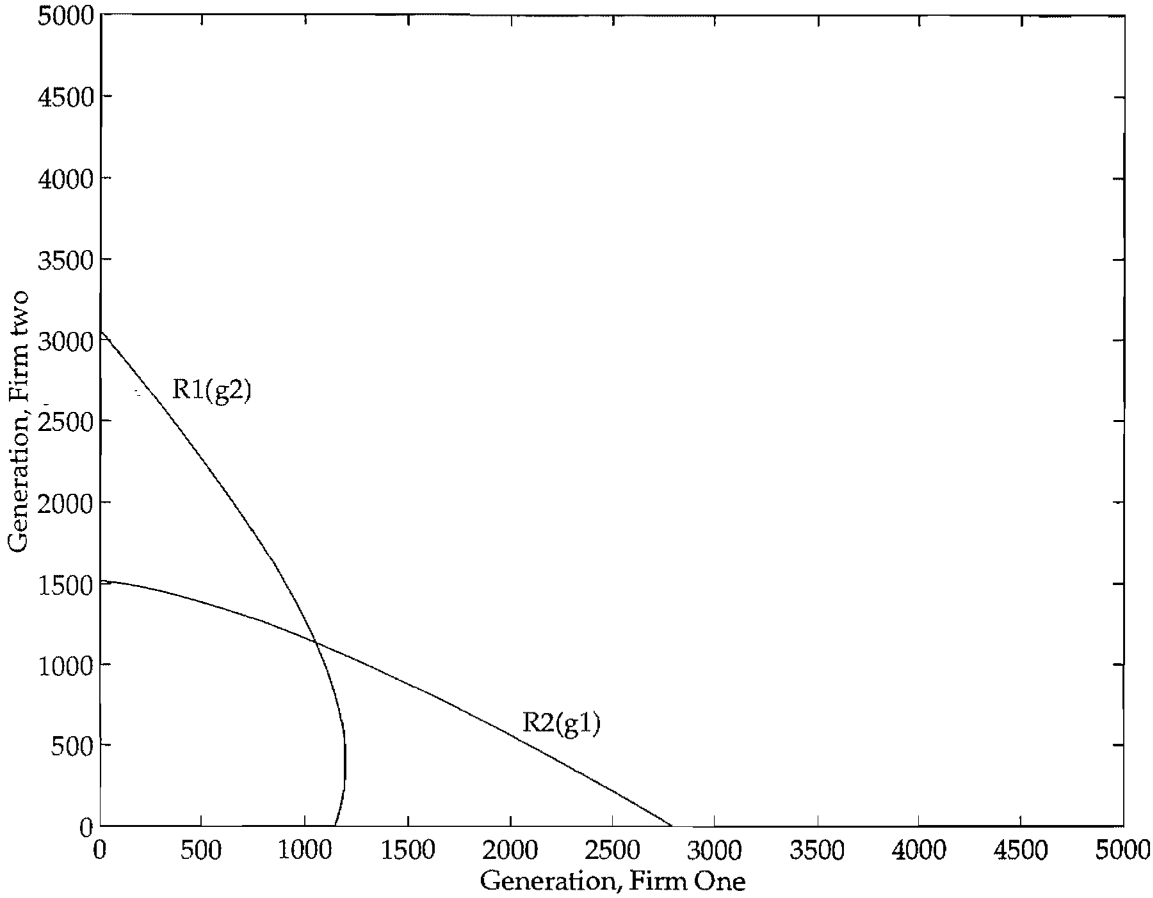

Figure 3.9: Reaction functions with constant elasticity of demand.

Figure 3.9 illustrates the reaction functions for the example case of two firms $j = \{1, 2\}$ with marginal costs of $\pi_1 = 1.5$ and $\pi_2 = 2.5$, and contract obligations of $k_1 = 800$ and $k_2 = 1200$ with demand curve parameters $p_0 = 3$, $\epsilon = -\frac{1}{3}$ and $g_0 = 2000$. The equilibrium point is where $g_1 = 1052$ and $g_2 = 1135$.

Theorem 3.3 considers firms whose marginal costs constant. Again the situation we wish to study is where the marginal cost function can be represented by a step curve, such as Figure 3.1. As before, we model this by constructing a separate reaction function for each section of our marginal cost curve, and switching from one to the next as the stations become fully utilised. The reaction functions for higher marginal costs will be closer to the origin than those for low marginal costs, and the lines joining the reaction functions will have slope of zero, so Tirole's condition will be satisfied. This is illustrated in Figure 3.10 for the example case of two firms $j = \{1, 2\}$ with marginal costs of $\pi_1 = 1.5$ for $g_1 < 700$ and $\pi_1 = 3.5$

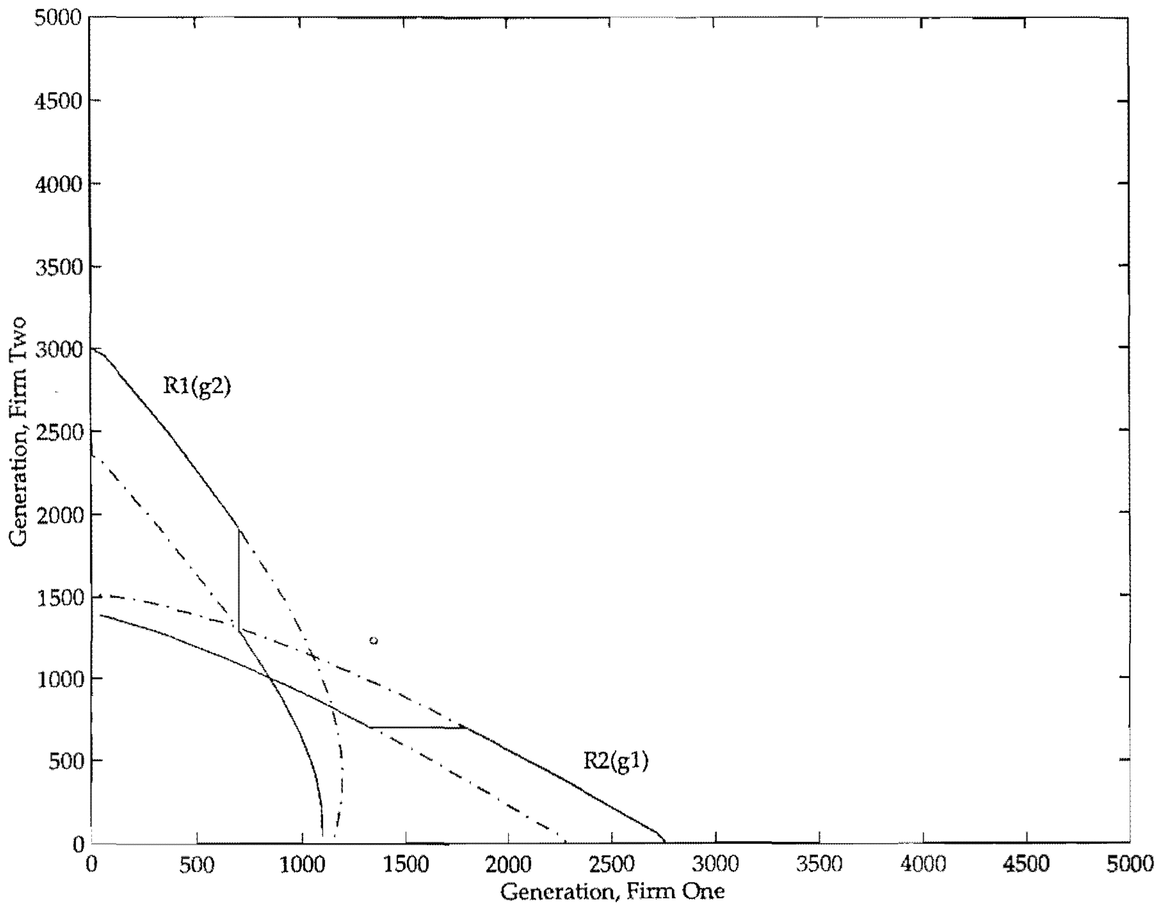

Figure 3.10: Composite reaction functions with constant elasticity of demand.

for $g_1 > 700$, $\pi_2 = 2.5$ for $g_2 < 1000$ and $\pi_2 = 5$ for $g_2 > 1000$, contract obligations of $k_1 = 800$ and $k_2 = 1200$, and demand curve parameters $p_0 = 3$, $\epsilon = -\frac{1}{3}$ and $g_0 = 2000$. The equilibrium point is now where $g_1 = 847$ and $g_2 = 1003$.

Theorem 3.4. *Consider the J firm Cournot market with profit functions*

$$z(\mathbf{g}) = p(\mathbf{g})[g_j - k_j] - \chi_j(g_j), \tag{3.97}$$

where $p(\mathbf{g})$ is inverse demand curve defining the market price for given levels of generation, and and $\chi_j(g_j)$ represents the production cost. If

$$p(\mathbf{g}) = p_0 \left[\frac{\sum_{n=1}^{J} g_n}{g_0}\right]^{\frac{1}{\epsilon}} \tag{3.98}$$

where ϵ, p_0 and g_0 are constants, $\chi_j(g_j)$ is a non-decreasing step function, and $R_j^{-1}(0) > R_i(0) = g_i^m$ (firm i's output that induces firm j to produce nothing exceeds firm i's monopoly *output), then there is a unique Cournot equilibrium vector of positive generation levels,* $\mathbf{g}$.

Proof. The proof follows that of Theorem 3.2. □

3.10 Conclusions

This chapter has presented a Cournot model of an electricity spot market. The existence of unique equilibria has been proved for both linear and constant elasticity demand functions for the situation where each of the firms has a monotone stepped marginal cost function. An efficient technique has been presented for locating this equilibrium point, and it is this technique which we use in our computer models. Some results and observations based on this single period model are presented in the following chapter. Chapter 5 discusses how we integrate this single period model into the multi-period model used for reservoir management.

Chapter 4

Single Period Behaviour

4.1 Introduction

In this chapter we consider the behaviour of electricity companies within some single period such as a half hour, or perhaps one sub-period of a LDC. We derive what we term *market response curves* (MRC) which describe how much the firms would offer to the market for a range of demand curves. These are similar to the traditional supply curve, although each point on the MRC is derived under different assumptions about the market demand curve.

We consider both linear and constant elasticity demand curves, with a range of elasticities, and a range of contract levels. We also compare results for wet and dry years, and consider the effect that the structure of the firms has on the MRC.

Finally we consider the situation where one of firms behaves as a perfect competitor, assuming it cannot affect the market price.

4.2 Market Response Curves

Klemperer & Meyer (1989) presented a model in which firms offer a supply curve to the market. At least for the implementations put forward by Green & Newbery (1992) and by Powell (1993), this has implied a single supply curve bid for the whole day. As discussed earlier, (Chapter 2), requiring a single supply curve to represent the responses to range of demands throughout the day is likely to introduce extra distortions. In the particular case of a hydro river chain, where marginal costs may well vary considerably throughout the day, a single supply curve may just not make sense. Our approach to this problem has been to model a separate Cournot game in each of several sub-periods throughout the day[1]. Within our single period model we determine the equilibria for each of the sub-periods, and combine these to give a total generation and average spot price for the period. In this Chapter, where we are interested in the market behaviour within a single time period, we use what we term *Market Response Curves* (MRCs) to show how the firms react to a range of demand curves. In many senses this may be thought of as an industry supply curve for the period, but it is really the Cournot equilibrium locus for a range of demand curves. In practice we have generated each point on the MRC by solving for the Cournot Nash equilibrium for each of a range of demand curves. This gives a price-quantity pair for each demand level[2]. A supply curve would give a quantity for a range of prices[3].

4.3 The Base Case

We begin with what we will refer to as the base case, *w34*, which is based on winter demand that would have been experienced in 1993 with the inflows that were experienced in 1934. For a start, we consider the breakup option we refer to as *ec2*, with stations allocated as described in Table 4.1. An interesting feature of this breakup option is that Firm One has both the major hydro capacity, and Huntly, which in practice is very often the marginal station in the NZ system. It has been suggested that keeping these assets together would enable the correct tradeoffs to be made between the hydro and Huntly, regardless of the practicalities of the market. If that is indeed the case, then we might well expect to see the operation

[1] In our implementation we usually consider the LDC to represent a week rather than a single day, but the principle is still the same.

[2] If we refer back to (3.75) we can see that while p_0 has a positive coefficient, g_j has a negative one. If we are to keep (3.76) in balance, then an increase in p_0 must be balanced by an increase in g_j. This means that our MRCs (which we construct by moving through an increasing series of demand curves, i.e. increasing p_0) will be monotonic-increasing.

[3] It is interesting to note that a subset of the points from the MRC do not form an optimal supply function to offer to the market for the period, in the sense of Klemperer & Meyer (1989). Each point is the solution to a distinct Cournot game, and as both Klemperer & Meyer (1989) and Green & Newbery (1992) state, a supply finction will only cross the Cournot line, once if at all. Our MRC, composed of many Cournot solutions, cannot be an equilibrium supply function. A supply function would be in response to a particular game, not a series of different games.

Station(s)	Capacity	Marginal Cost	Owner
Waitaki and Waikato[4]	2455	2.0	Firm 1
Huntly	900	2.0	Firm 1
New Plymouth	518	2.5	Firm 2
Stratford	178	3.5	Firm 2
Marsden A	103	6.0	Firm 2
Otahuhu	81	11.0	Firm 2
Whirinaki	194	15.0	Firm 2
Clutha[5]	244	0.0	Firm 2
Manapouri[6]	570	0.0	Firm 1

Table 4.1: Break-up option *ec2*.

under the *ec2* option to be closer to the perfectly competitive outcome than other options where this feature is absent.

4.3.1 How We Define Contracts

Before presenting the example results, we must first explain some of our notation. In what follows we will refer to two types of contracts, *two-way options* and *one-way options*, commonly known as *call options*. We will typically refer to the former as the level of contracting, and to the latter as the level of backup. Where this distinction is not obvious more precise terms will be used.

We typically define the level of contracting as a percentage of the load that the firm would be expected to generate under PC. The contracts are set as a percentage of that expected generation level. We commonly use the term *fully contracted* to mean that a firm has sold contracts for exactly 100% of its expected PC generation[6].

In our model the firms are allowed to buy backup contracts from each other, but not from the demand side or the Fringe[7]. These backup contracts transfer some

[5]Since we only develop a single reservoir model in this thesis, the Waitaki and Waikato hydro systems are aggregated.

[5]Both Clutha and Manapouri are here modelled as *run of river* stations, with capacities set to their average winter output.

[6]Hydro inflows, in particular, will cause a major variation of the actual PC levels in any given year. This means that 100% contracting based on a particular expected level will not exactly match the PC levels for any given year.

[7]One could easily construct a financially equivalent model where instead of having back-up

of a firm's obligation to the other firm if the spot price rises above the strike price. We have again defined these as a percentage, this time of the amount of backup required to return Firm Two to full contracting[8]. In our computational experiments we have defined back-up contracts with strike prices at each marginal fuel cost.

4.4 Constant Elasticity Demand Curves

In this section we assume that the market demand at a given moment can be described by a constant elasticity demand curve. We begin by considering the case where the elasticity of demand is $-\frac{1}{3}$.

Figure 4.1 shows the market response for the case where both firms are 100% contracted, and the level of backup is also 100%[9]. The market generation levels, marked with a +, fit closely to the step curve that is the PC supply curve (the marginal cost curve[10]). The reason for this is simple: at each point the contracts match the PC generation level exactly. The lower graph in these figures shows the division between the two firms. With no back-up, each firm's generation is non-decreasing with increasing total generation.

Compare this with Figure 4.2 where the firms are both only 50% contracted, and there is no backup. Here we see a significant move away from the PC levels, with the spot price being between 20 and 50% higher than PC. The reason for this shift is clear if we look back to (3.12) which tells us that the market level will lie somewhere between the PC level and the contract level.

Such a shift is extreme, to say the least, but it also seems unlikely in light of both overseas experience (Green & Newbery 1992, Wiedswang 1993), and the current

contracts with another firm, the firms had one way contracts with the consumers, and the consumers bought contracts from one firm for up to a certain price, and from the other firm for above that price.

[8]This was a somewhat arbitrary choice, to base back-up contracts on the idea of returning Firm Two to full contracting. Our motivation was the idea that the hydro stations would buy back-up from the thermal stations to cover themselves for inflow variations. A more accurate scheme would be to enter contracts as actual quantities rather than percentages, but in the absence of realistic data on contracting we chose to work with percentages. This does have the advantage that it better enables comparison across models and market structures.

[9]Note that backup is still required since the contracts for Firm One were based on the expected hydro generation which, depending upon the inflow, will probably not be the actual capacity.

[10]As we discuss later in this Chapter, it is arguable that short run marginal cost data is not appropriate for use in the simulation model, just as short run demand elasticity may not be appropriate.

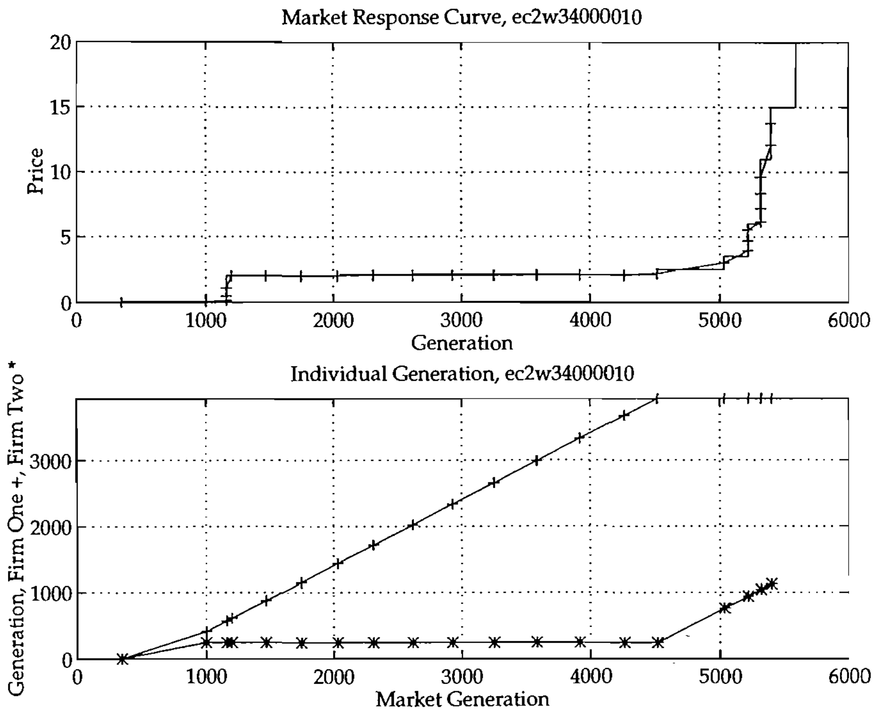

Figure 4.1: Market Response Curve and individual generation for base case, both Firms 100% contracted, full backup.

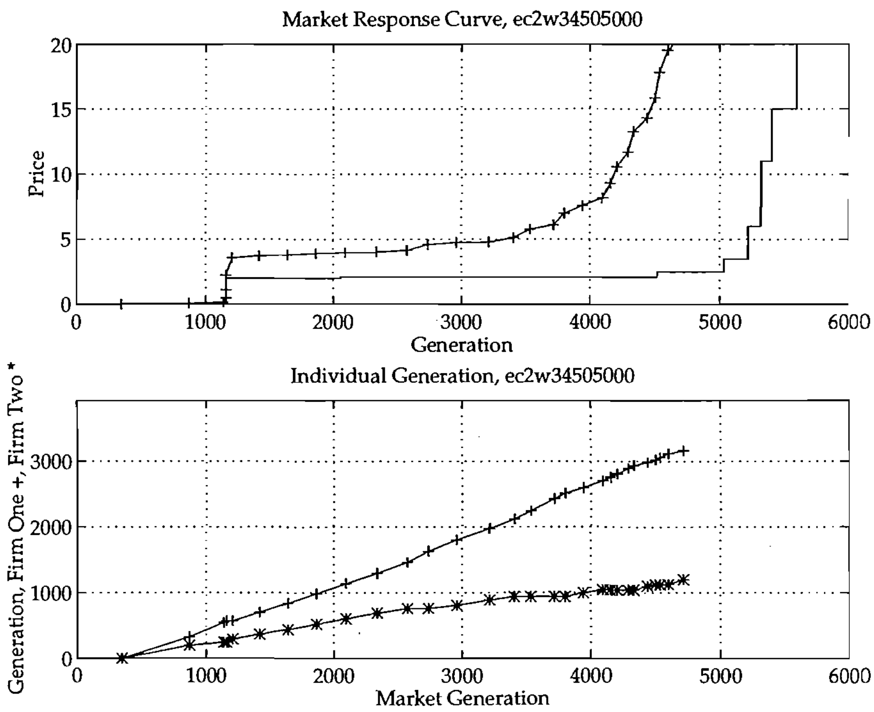

Figure 4.2: Market Response Curve and individual generation, both Firms 50% contracted, no backup.

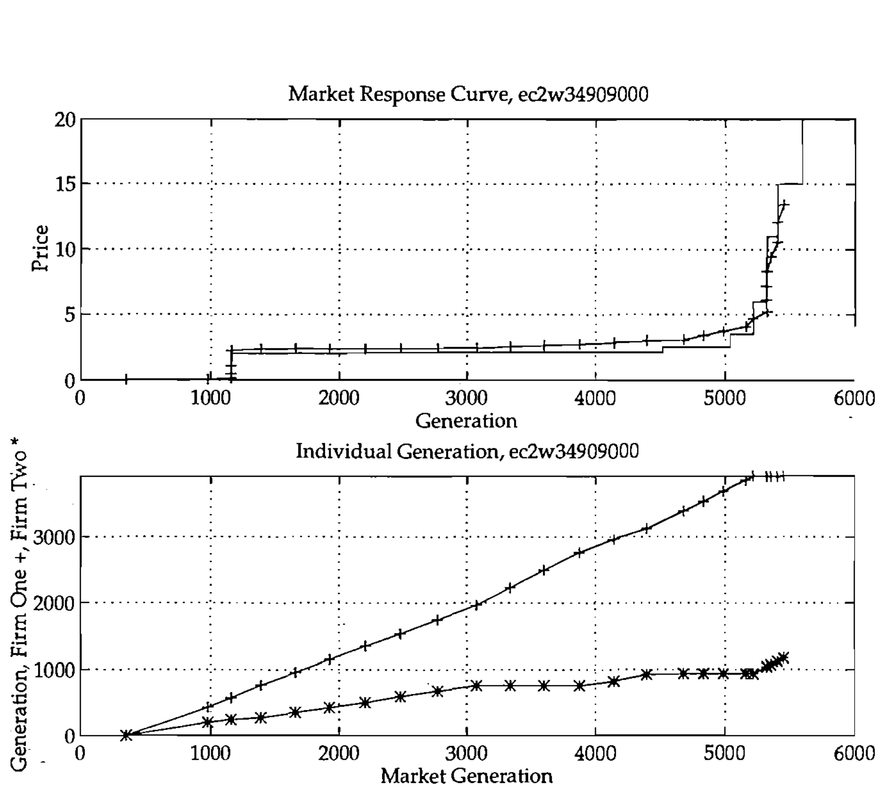

Figure 4.3: Market Response Curve and individual generation, both Firms 90% contracted, no backup.

situation in New Zealand, where long term contracts have been offered up for to 87% of the dry year capacity[11]. A more likely scenario is that shown in Figure 4.3 where the firms are both 90% contracted, although there is no backup. The MRC is much closer to the PC curve, although the spot price is at times up to 25% higher than PC. Note that this does not imply a 25% rise in electricity bills for the consumers, as the 25% is only on the 10% of demand that isn't contracted[12].

The effect of backup contracts is not always positive, as can be seen when we compare Figure 4.2 with Figure 4.4, the latter having 100% backup. Not only is there a clear shift in the response curve, but also the lower graph shows there is a significant change in the relative outputs of the two firms, with Firm Two taking on more of the generation as they face greater contracts via the backup arrangements they have offered Firm One. The perhaps counter-intuitive[13] thing about

[11]This is for ECNZ. The contracts for Contact, ECNZ's main competitor are not public knowledge.

[12]As mentioned in footnote 14 the contract price may reasonably be expected to converge on the long run average spot price.

[13]The purpose of the back-up contracts is to restore merit order in production, that is, to improve production efficiency. The downside of this is in allocative efficiency. The back-up contracts

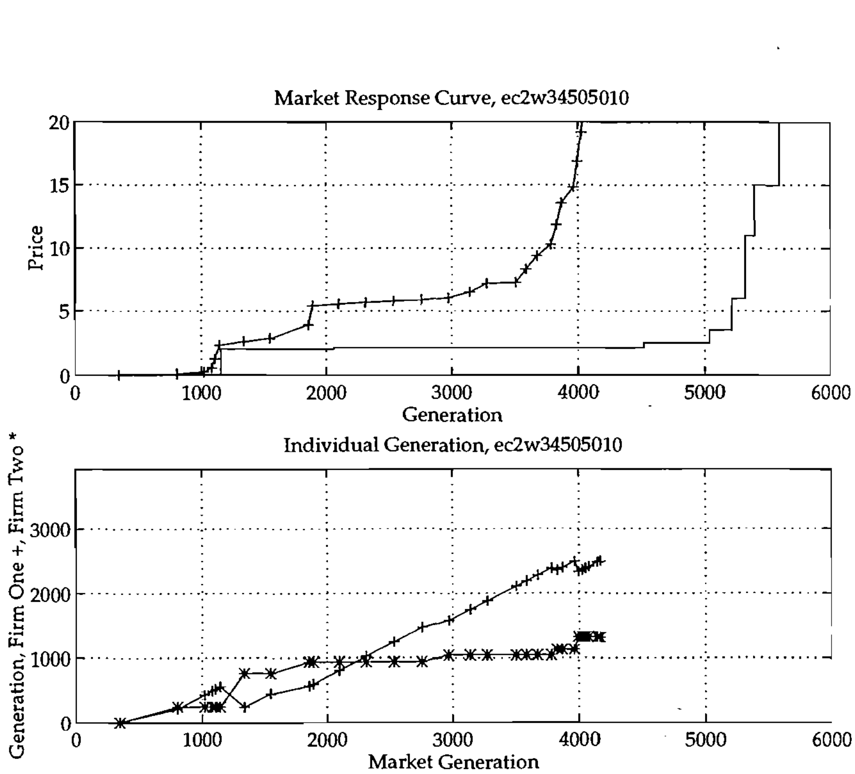

Figure 4.4: Market Response Curve and individual generation, both Firms 50% contracted, 100% backup.

this comparison is that in the situation where there is greater backup, prices are higher and output is lower. The explanation is simple. Firm One is the dominant firm, with 3925 MW compared with 1318 MW in Firm Two. Firm One has only low merit order stations, so as the price rises, the back-up contracts will be transferring the contract obligations away from Firm One, to Firm Two. Hence the dominant firm is now more lightly contracted, and will act more strongly to push the spot price up. Firm two will act against this, as they are now more heavily contracted, but being the smaller firm they have less influence, and the spot price will inevitably rise.

Higher back-up does not always imply lower distortion, rather appropriate back-up can reduce distortion. This is largely a matter of definition. We have defined 100% back-up as the amount required to adjust Firm Two's contracts to the amount they would expect to generate under PC. With this definition when both firms are 50% contracted 100% back-up implies an unrealistic level, in that

have an effect not unlike that of collusion, increasing the monopoly powers of the suppliers, and re-allocating wealth from consumers to suppliers. It is this effect on allocative efficiency that we observe here.

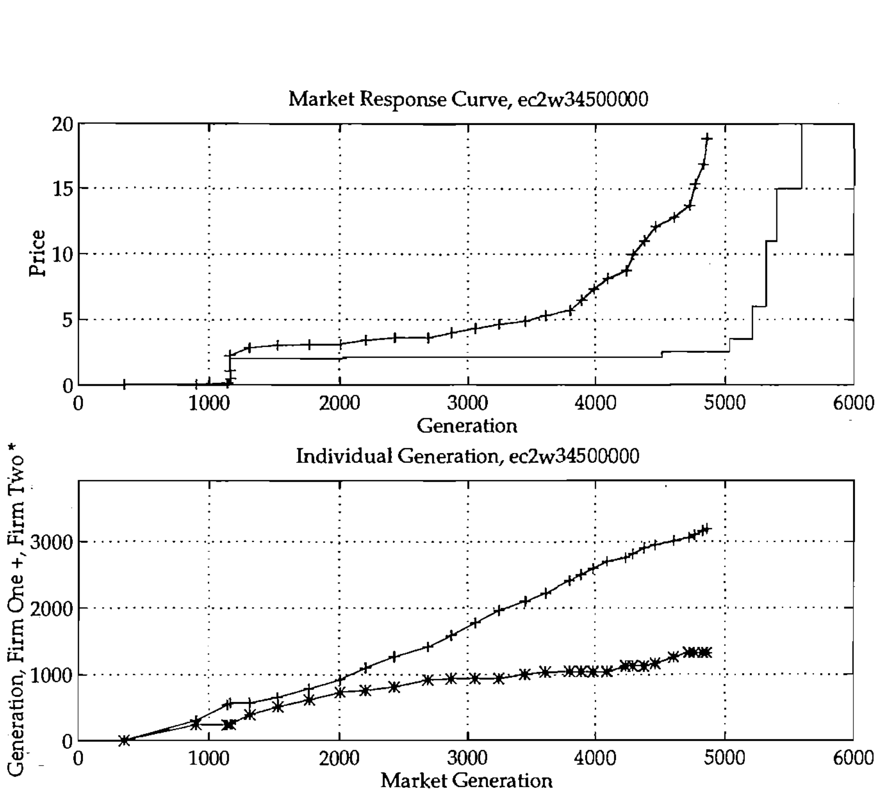

Figure 4.5: Market Response Curve and individual generation, Firm One 50% contracted, Firm Two 100% contracted, no backup.

it returns Firm Two to 100% contracting, when their intention was only to be 50% contracted. Full back-up in this instance would perhaps be more appropriately be defined as the level which would return Firm Two to 50% of their PC production.

To further illustrate this idea of market power, consider Figure 4.5 where Firm One is 50% contracted, and Firm Two is 100% contracted, with no back-up. The response curve shows significantly higher prices and lower quantities than the PC levels, for all levels of demand. Compare this to Figure 4.6 in which we have the reverse situation, with Firm One 100% contracted, and Firm Two 50% contracted, again with no back-up. Here the response curve is very close to the PC curve at all levels of demand. There are two reasons for the differences.

The first is to do with the overall level of contracting in the market. Recall that Firm One is much larger than Firm Two, and so 50% of Firm One's contracts is a much greater amount than 50% of Firm Two's. The overall level of contracting is then much greater in Figure 4.6 than in Figure 4.5, and so the market response is much closer to PC.

The second reason is to do with market power at different demand levels. Since

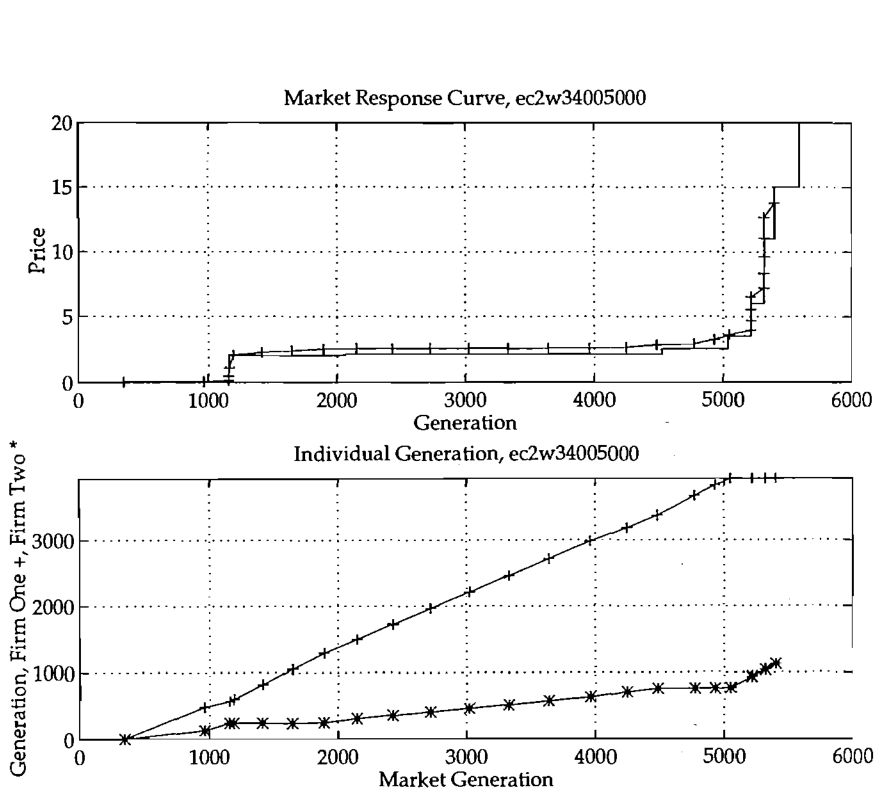

Figure 4.6: Market Response Curve and individual generation, Firm One 100% contracted, Firm Two 50% contracted, no backup.

Firm One has the large, low merit order stations, it can influence the market at all levels of demand. Firm Two, on the other hand, has the high merit order stations, and cannot influence the spot market at low prices without suffering substantial marginal (financial) losses. This is further illustrated if we look at Figure 4.7, which is similar to Figure 4.5, except that the marginal cost of hydro is set to 10 cents, which places it much higher up the merit order. This situation could well occur in a dry year, for example, when the marginal water value would be substantially higher. The result is rather interesting, with the market response curve now having sections both under and over the PC levels.

Alternatively, we can consider a different breakup option which we call *ww*, as described in Table 4.2. This option has Firm One owning just the major hydro storage reservoirs, Waitaki and Waikato. Firm Two owns Huntly, the major thermal station at 2 cents, and Clutha and Manapouri are owned by the Fringe. Figure 4.8 is similar to Figure 4.5, but for this *ww* breakup option. The MRC is much closer to PC since Firm Two can now influence the spot price at relatively low levels of output. This may seem contrary to the expectations that having Huntly in

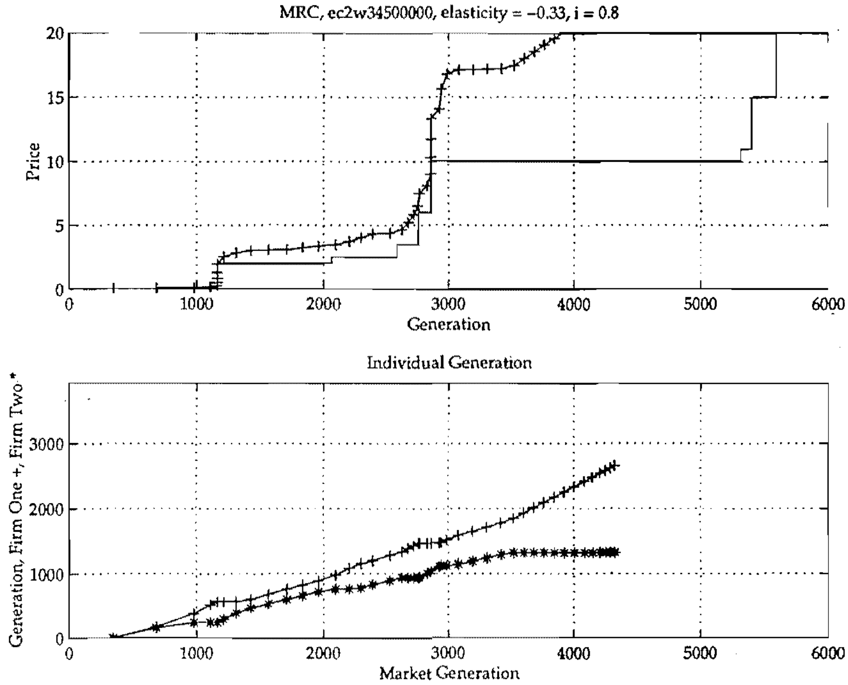

Figure 4.7: Market Response Curve and individual generation, Firm One 50% contracted, Firm Two 100% contracted, no backup, marginal water value = 10.

the same firm as the major hydro stations would encourage better coordination of Huntly and the hydro resources. However the real test of this is in the medium term model, where the marginal water value changes throughout the year. This is discussed further in Chapter 6.

In this section we have demonstrated the major impact that contracts have on generation levels. Low levels of overall contracting lead to low levels of output, and high levels of contracting lead to high output. The effect of back-up contracts is to increase the coordination efficiency between the firms, but may be at some detriment to the allocative efficiency. That is, while the overall fuel cost for the market as a whole may be reduced, there may be some transfer of wealth from consumers to the producers, who will have greater market power as a result of the back-up contracts. We have shown how the portfolio of plant that a supplier holds influences their market power, and ultimately the level of price distortion in the spot market as a whole. It is worth noting that letting an under-contracted, low merit order company buy back-up may well increase the price distortions rather than reduce them.

Station(s)	Capacity	Marginal Cost	Owner
Waitaki and Waikato	2455	2.0	Firm 1
Huntly	900	2.0	Firm 2
New Plymouth	518	2.5	Firm 2
Stratford	178	3.5	Firm 2
Marsden A	103	6.0	Firm 2
Otahuhu	81	11.0	Firm 2
Whirinaki	194	15.0	Firm 2
Clutha	244	0.0	Fringe
Manapouri	570	0.0	Fringe

Table 4.2: Break-up option *ww*.

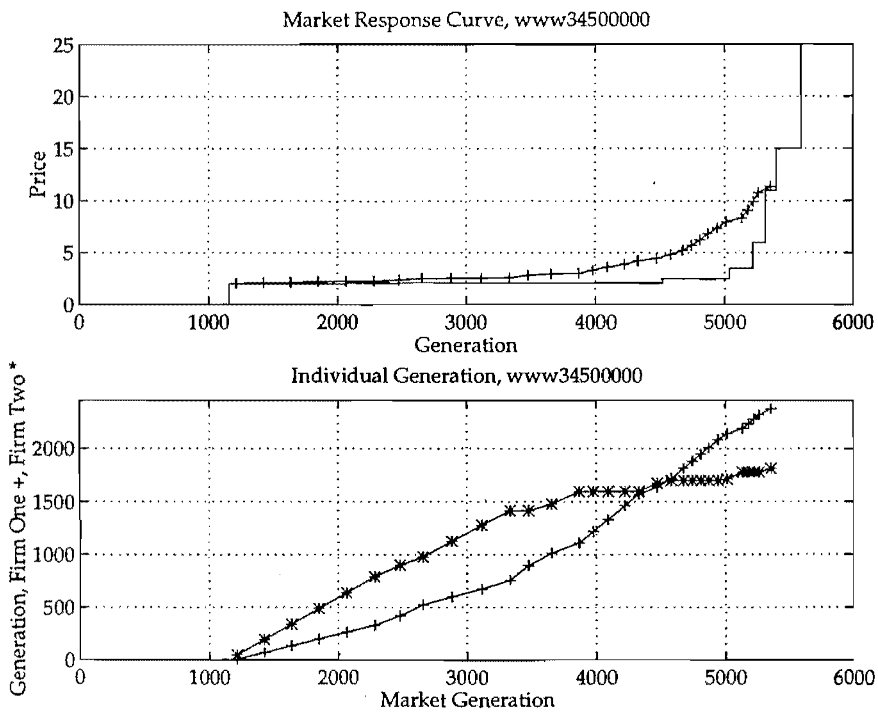

Figure 4.8: Market Response Curve and individual generation for breakup option *ww*, Firm One 50% contracted, Firm Two 100% contracted, no backup.

4.4.1 Different Levels of Demand Elasticity

A complication for a model such as ours is that it is not entirely clear whether we should be using short run or long run elasticities. In reality, while peoples' response to price changes may be small in the short term, they will be greater in the medium term, with a change in usage patterns, and switching to alternative fuels. Ideally we would model this with a low short run elasticity and a higher medium term elasticity. In practice our model only lets us set one elasticity, so we compromise with a value between the estimates for the short and medium term. We believe this is appropriate since the owners will undoubtedly have their focus on the mid-term and long-term[14] response when playing this repeated game[15]. With that in mind, it is obviously important to consider the sensitivity of our results to the choice of elasticity, and that is what we concentrate on in this section. It should be kept in mind, though, when reading through this section, that the manager must ultimately make that compromise, and the elasticity would be ultimately chosen to give the most satisfactory results for the medium term simulation reported later.

We will consider elasticities in the range from -0.1 to -0.8, which cover the extremes reported elsewhere (Borenstein & Bushnell 1997, Green & Newbery 1992). There is little point in presenting the results for the base case with full contracting, as the response curve will again exactly match the PC curve. Obviously the most dramatic changes will be for the cases where the distortion away from PC is the greatest, that is, extreme over or under contracting. But even for the case of both firms contracted at 90% and with no backup the MRC is quite sensitive to changes in elasticity. Figure 4.9, which is similar to Figure 4.3, shows the market response for demand elasticities of -0.1, -0.33 and -0.8. For the most extreme of these three cases, elasticity of -0.1, the response curve is about as far from PC as in Figure 4.2 where the elasticity was -0.33, but the contracts were at 50%.

From these results it is obvious that our model is indeed sensitive to changes in demand elasticity, and some caution should be used when deciding upon the

[14]A related issue, but one we do not consider in this Thesis is the idea that energy spot market prices will feed forward to future contract prices. Thus raising spot prices may well lead to even greater profits in the future aqs contract prices increase also. See also Batstone (1997).

[15]The discerning owner will realise that while extra high profits may be made on any given day, such levels could not be sustained without either enticing new entry or encouraging consumers to find substitutes for electricity.

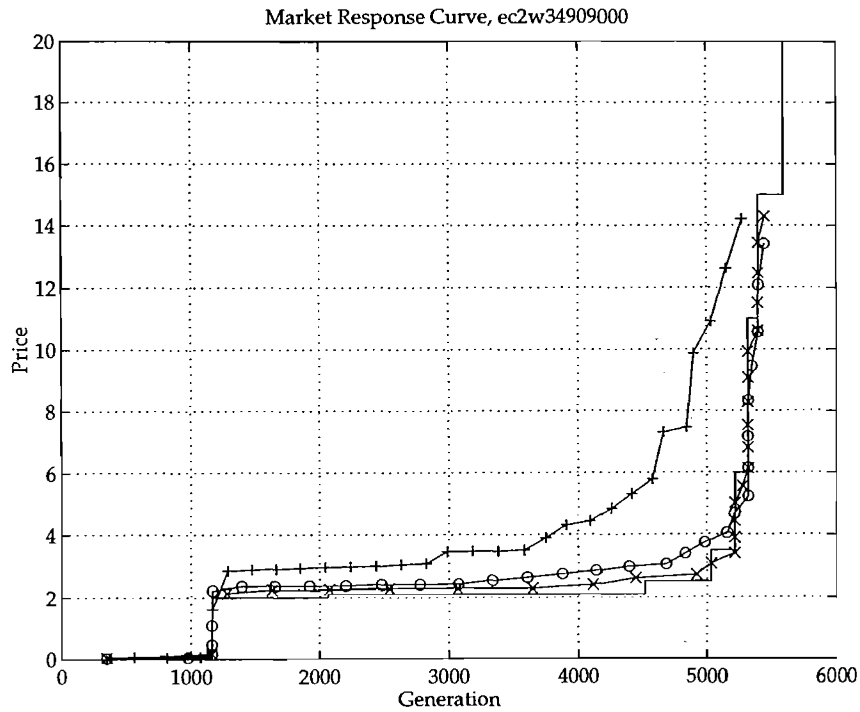

Figure 4.9: Market Response Curve, both Firms 90% contracted, no backup. Elasticity of demand at -0.1 (+), -0.33 (o), and -0.8 (x).

appropriate elasticity.

4.5 Linear Demand Curves

Here we assume that the market demand can be described by a linear demand curve, and look at the resulting equilibrium conditions. As with the previous section we are interested in how sensitive our model is to the choice of demand curve. Figure 4.10 shows the MRC with the slope of the demand curves set to $-1/500$, with both firms 50% contracted, and no backup. This equates roughly to Figure 4.2. As we had hoped, our model produces similar results for the linear demand curves to those for the constant elasticity demand curves. Figure 4.11 shows the sensitivity of the model to change in slope of demand, similar to that shown for the constant elasticity curves in Figure 4.9. However, for a given linear demand curve tangent to a constant elasticity demand curve the price will be lower at all levels except the tangent point. This is illustrated in Figure 4.12.

Again the model is very sensitive to changes in the slope of the demand curve.

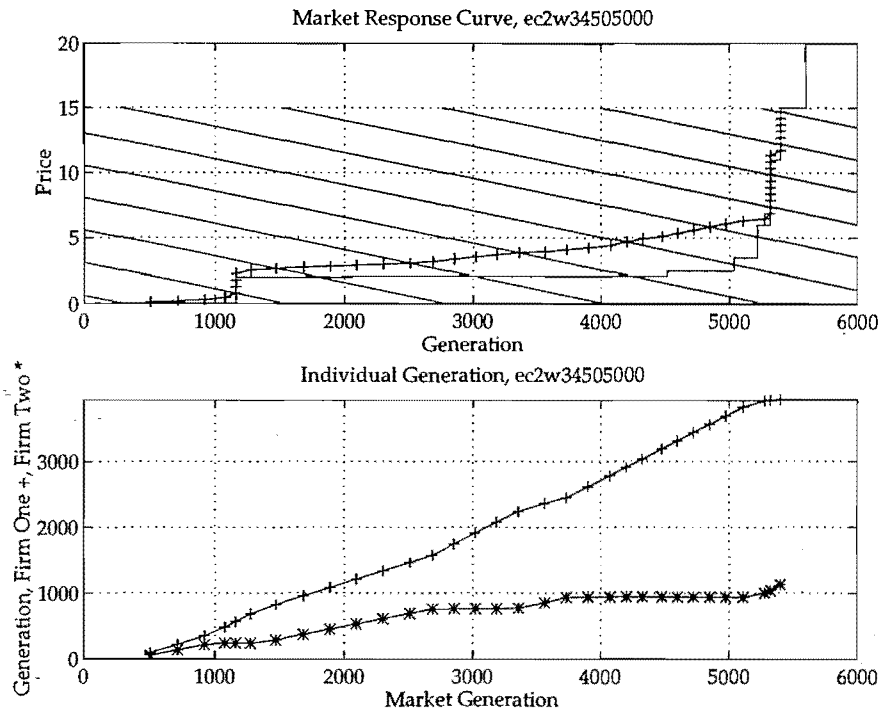

Figure 4.10: Market Response Curve and individual generation, both Firms 50% contracted, no backup.

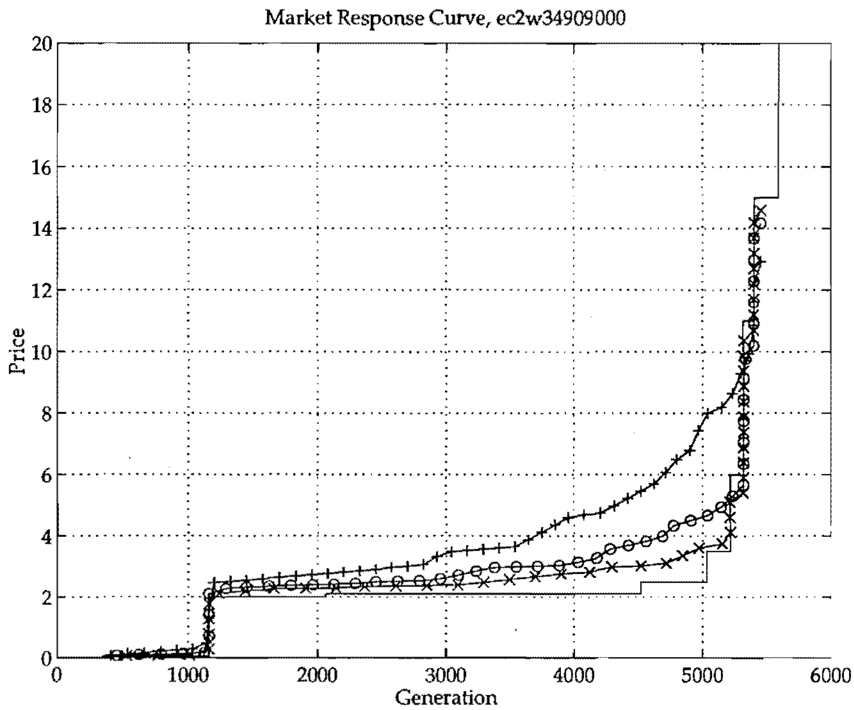

Figure 4.11: Market Response Curve, both Firms 90% contracted, no backup. Slope of demand at $-\frac{1}{100}$ (+), $-\frac{1}{250}$ (o), and $-\frac{1}{500}$ (x).

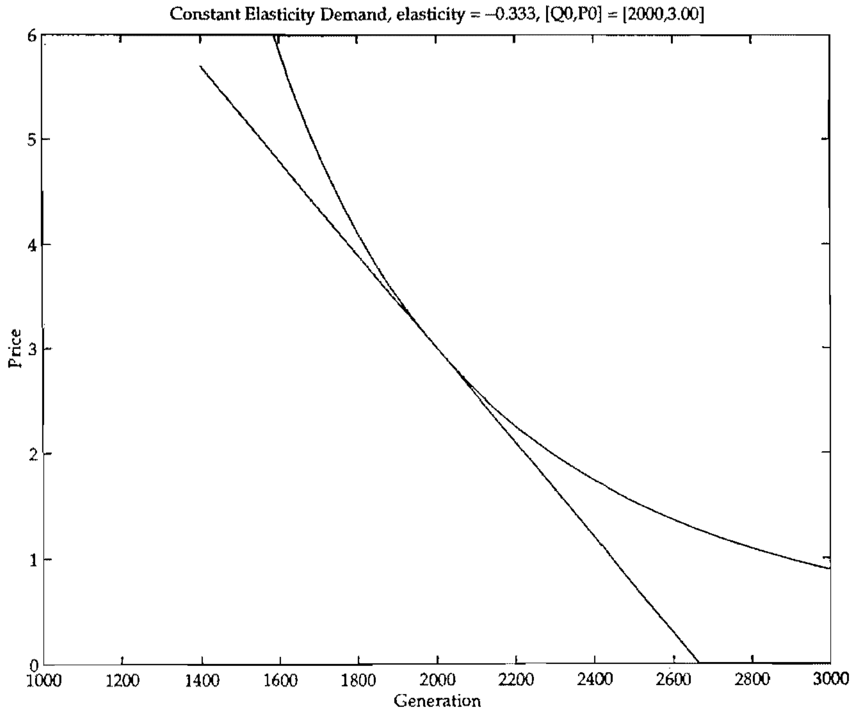

Figure 4.12: Constant elasticity demand curve is always above the tangent linear demand curve.

It seems that with appropriate choice of slope the results using linear demand curves are comparable with those using constant elasticity demand curves, especially if the equilibrium point is close to the reference point for the demand curve. However, as shown in Figure 4.12, the discrepancy becomes greater as the equilibrium moves away from the reference point. When choosing between a linear and a constant elasticity representation we should, then, consider how close to our point estimates the equilibrium is likely to be. In terms of efficiency the linear demand curve allows closed form solution, whereas the constant elasticity demand curve requires solution by numerical methods.

4.6 Summary Tables

The following tables summarise the distortion in the market as measured by our *Price Distortion Index* (PDI). The PDI is defined as the ratio of the area under the MRC minus the area under the PC supply curve to the area under the PC supply curve. As examples, the PDI for Figure 4.1 is −0.01 and for Figure 4.2 is 2.13. For any particular situation a better measure of the price distortion would probably be given by a weighted index, with weights based on the relevant LDC, but for general comparisons of the sort we wish to make here this unweighted index seems more appropriate. A negative PDI simply implies that the MRC was lower than the PC supply curve.

The tables emphasise the points made elsewhere in this chapter. In particular distortion is higher for lower levels of contracting, and for less elastic demand. Corresponding to each table is a graph, Figure 4.13–Figure 4.21, of the same data. In the graphs the PDI is plotted against the mean total level of contracting[16].

While there is a clear trend in the graphs, showing the PDI decreases as the overall level of contracting increases, there are some irregularities[17]. The explanation for these is that we have used an imperfect measure of the total contracting level, namely the mean across the MRC. If we had plotted the graphs for a particular demand curve, rather than a range of demand curves, the trends would have been more obvious. However a single demand curve might not give as good a representation of the overall distortions in the market.

It is clear from comparing Figure 4.13,Figure 4.14 and Figure 4.15 that there is greater price distortion for the less elastic cases. It is not so clear cut as to the effect of back-up contracts as we have implemented them. While increasing back-up seems to have helped for high levels of contracting, it has had quite the opposite effect for low levels of contracting. But compare Figure 4.14 (no back-up) with Figure 4.20 (100% back-up). The rows across Figure 4.14 correspond with the rows of Table 4.4, where Firm One's contracts are constant, and Firm Two's are changing.

[16]Recall that Firm One is roughly twice the size of Firm Two, and so 50% of Firm One's capacity is a rather different number to 50% of Firm Two's capacity. It seems more reasonable to expect the PDI to be correlated to the total contracting amount, rather than to a percentage of either firms' capacity. However the PDI is based on the MRC, which is plotted for a range of market demand levels, and a corresponding range of contract amounts. But just as we have condensed the distortion measures into a single PDI, we have also condensed the range of contract amounts (all corresponding to the same percentage, but of differing demands) into the mean contract amount. It is this value which we refer to as the mean total contract amount in the graphs.

[17]Figure 4.19 has some irregularities due to missing data. The missing data arose from the situation where the total contracting was sufficiently low that the back-up contracting resulted in one or the other company having a negative amount on contract. While there is nothing inherently incorrect about a negative contract amount, it would normally be expected from the demand side, not the supply side. Our program has not been set up to cope with this situation, as it does not really fall within the range of levels we expected to study.

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	56.87	36.68	23.13	17.64	14.57	12.77	11.63	10.87
60%	20.41	14.91	10.41	8.26	6.94	6.16	5.57	5.20
70%	8.76	7.01	5.36	4.33	3.69	3.24	2.92	2.67
80%	4.22	3.47	2.95	2.41	2.04	1.78	1.58	1.44
90%	2.05	1.74	1.47	1.26	1.06	0.89	0.77	0.66
100%	0.94	0.72	0.52	0.40	0.26	0.14	0.04	-0.04
110%	0.39	0.23	0.11	0.02	-0.07	-0.16	-0.22	-0.26
120%	0.13	-0.01	-0.10	-0.16	-0.23	-0.30	-0.34	-0.38

Table 4.3: PDI. No back-up, elasticity of demand -0.1

In Figure 4.20 these horizontal bands are absent, and there is a monotonic relationship between total contracting and PDI. It is the effect of the back-up contracts which produces the difference between the figures. In the case where there is no back-up the contracting of Firm One (the larger firm) has a much greater influence on the PDI than the contracting of Firm Two. When there is back-up, the contracts are re-allocated amongst the firms, and it is no longer important which firm sold the original contracts. If either firm increases contracting by a particular quantity, then the PDI will drop by the same amount.

The message once again is that back-up is improving productive efficiency, but not necessarily allocative efficiency. When contracting is low, and monopoly power is correspondingly high, back-up increases market power, and increases price distortions. When contracting is high, and monopoly power correspondingly low, back-up increases productive efficiency and this reduces price distortion.

4.7 The Monopolist vs the Perfect Competitor

Let us now turn our attention to the situation where only one of the two firms is acting as an oligopolist, with the other being a price-taker, and generating whenever its marginal cost is less than the spot price, regardless of contract obligations. (Equivalently, each station is owned by separate price taker.) In these examples Firm One, the one controlling the hydro reservoirs, is the monopolist, and Firm Two is the PC.

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	2.13	2.14	2.08	1.94	1.84	1.75	1.68	1.60
60%	1.50	1.41	1.41	1.40	1.34	1.27	1.21	1.16
70%	0.84	0.80	0.74	0.71	0.61	0.52	0.43	0.40
80%	0.50	0.46	0.41	0.38	0.31	0.24	0.19	0.16
90%	0.32	0.28	0.23	0.21	0.16	0.09	0.06	0.03
100%	0.20	0.17	0.12	0.10	0.06	0.01	-0.03	-0.05
110%	0.12	0.09	0.04	0.02	-0.02	-0.06	-0.08	-0.11
120%	0.06	0.03	-0.01	-0.03	-0.07	-0.11	-0.13	-0.15

Table 4.4: PDI. No back-up, elasticity of demand -0.3

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	0.41	0.39	0.37	0.36	0.32	0.26	0.23	0.22
60%	0.32	0.30	0.28	0.27	0.23	0.19	0.16	0.15
70%	0.25	0.23	0.21	0.19	0.16	0.13	0.10	0.09
80%	0.19	0.17	0.15	0.14	0.11	0.08	0.06	0.05
90%	0.15	0.13	0.11	0.10	0.07	0.05	0.03	0.02
100%	0.12	0.09	0.08	0.06	0.04	0.02	0.00	-0.01
110%	0.08	0.06	0.05	0.03	0.01	-0.01	-0.02	-0.03
120%	0.05	0.03	0.02	0.00	-0.02	-0.03	-0.05	-0.05

Table 4.5: PDI. No back-up, elasticity of demand -0.8

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	59.22	40.89	29.99	23.05	17.88	14.06	11.30	9.05
60%	23.14	17.06	13.05	10.18	8.02	6.32	4.97	4.04
70%	10.56	8.03	6.13	4.83	3.89	3.09	2.50	1.92
80%	5.17	3.94	3.04	2.43	1.94	1.52	1.18	0.92
90%	2.55	1.96	1.54	1.20	0.83	0.57	0.28	0.08
100%	1.13	0.78	0.50	0.30	0.14	-0.01	-0.11	-0.18
110%	0.37	0.22	0.10	-0.00	-0.09	-0.16	-0.22	-0.28
120%	0.07	-0.01	-0.08	-0.14	-0.20	-0.26	-0.31	-0.35

Table 4.6: PDI. 50% back-up, elasticity of demand -0.1

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	2.38	2.12	1.95	1.81	1.66	1.52	1.38	1.27
60%	1.76	1.62	1.43	1.28	1.09	0.87	0.66	0.51
70%	1.10	0.89	0.73	0.61	0.51	0.41	0.32	0.23
80%	0.20	0.14	0.09	0.04	-0.00	-0.04	-0.04	-0.05
90%	0.54	0.46	0.39	0.33	0.29	0.21	0.14	0.08
90%	0.33	0.27	0.22	0.18	0.14	0.09	0.03	-0.01
100%	0.20	0.15	0.12	0.08	0.05	0.02	-0.02	-0.06
110%	0.11	0.07	0.04	0.02	-0.00	-0.03	-0.06	-0.10
120%	0.04	0.01	-0.01	-0.03	-0.05	-0.08	-0.10	-0.13

Table 4.7: PDI. 50% back-up, elasticity of demand -0.3

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	0.42	0.39	0.36	0.33	0.31	0.27	0.22	0.17
60%	0.32	0.30	0.27	0.24	0.21	0.18	0.15	0.12
70%	0.24	0.22	0.20	0.18	0.16	0.13	0.10	0.08
80%	0.18	0.17	0.15	0.13	0.11	0.08	0.06	0.04
90%	0.14	0.12	0.11	0.09	0.07	0.06	0.04	0.02
100%	0.10	0.09	0.07	0.06	0.04	0.03	0.01	-0.01
110%	0.06	0.05	0.04	0.03	0.01	-0.00	-0.02	-0.03
120%	0.03	0.02	0.01	-0.00	-0.02	-0.03	-0.04	-0.06

Table 4.8: PDI. 50% back-up, elasticity of demand -0.8

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	177.61	106.26	64.79	40.49	26.24	17.09	11.54	7.98
60%	57.00	35.84	23.31	15.23	10.35	7.19	4.88	3.37
70%	20.71	13.60	9.30	6.46	4.40	3.03	2.09	1.41
80%	8.56	5.83	3.97	2.73	1.88	1.27	0.79	0.37
90%	3.60	2.49	1.70	1.14	0.66	0.31	0.13	0.01
100%	1.52	1.00	0.53	0.26	0.10	-0.01	-0.09	-0.15
110%	0.43	0.21	0.06	-0.03	-0.11	-0.17	-0.22	-0.26
120%	0.03	-0.06	-0.13	-0.18	-0.23	-0.27	-0.30	-0.33

Table 4.9: PDI. 100% back-up, elasticity of demand -0.1

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	2.49	2.17	1.90	1.65	1.44	1.22	0.95	0.64
60%	1.83	1.59	1.38	1.16	0.83	0.59	0.45	0.34
70%	1.32	1.09	0.75	0.55	0.42	0.32	0.24	0.18
80%	0.68	0.51	0.39	0.30	0.22	0.16	0.11	0.07
90%	0.36	0.28	0.21	0.15	0.10	0.06	0.03	0.00
100%	0.19	0.14	0.09	0.05	0.02	-0.01	-0.03	-0.06
110%	0.08	0.04	0.01	-0.01	-0.04	-0.06	-0.08	-0.10
120%	0.01	-0.02	-0.04	-0.07	-0.09	-0.11	-0.12	-0.14

Table 4.10: PDI. 100% back-up, elasticity of demand -0.3

Contracting	Firm Two							
Firm One	50%	60%	70%	80%	90%	100%	110%	120%
50%	0.47	0.41	0.35	0.31	0.27	0.24	0.21	0.18
60%	0.34	0.30	0.26	0.23	0.20	0.17	0.14	0.12
70%	0.25	0.22	0.19	0.16	0.14	0.12	0.10	0.08
80%	0.18	0.15	0.13	0.11	0.09	0.08	0.07	0.05
90%	0.13	0.11	0.09	0.08	0.06	0.05	0.04	0.03
100%	0.08	0.07	0.06	0.05	0.03	0.02	0.01	-0.00
110%	0.06	0.04	0.03	0.02	0.01	-0.00	-0.01	-0.02
120%	0.03	0.02	0.00	-0.01	-0.01	-0.02	-0.03	-0.04

Table 4.11: PDI. 100% back-up, elasticity of demand -0.8

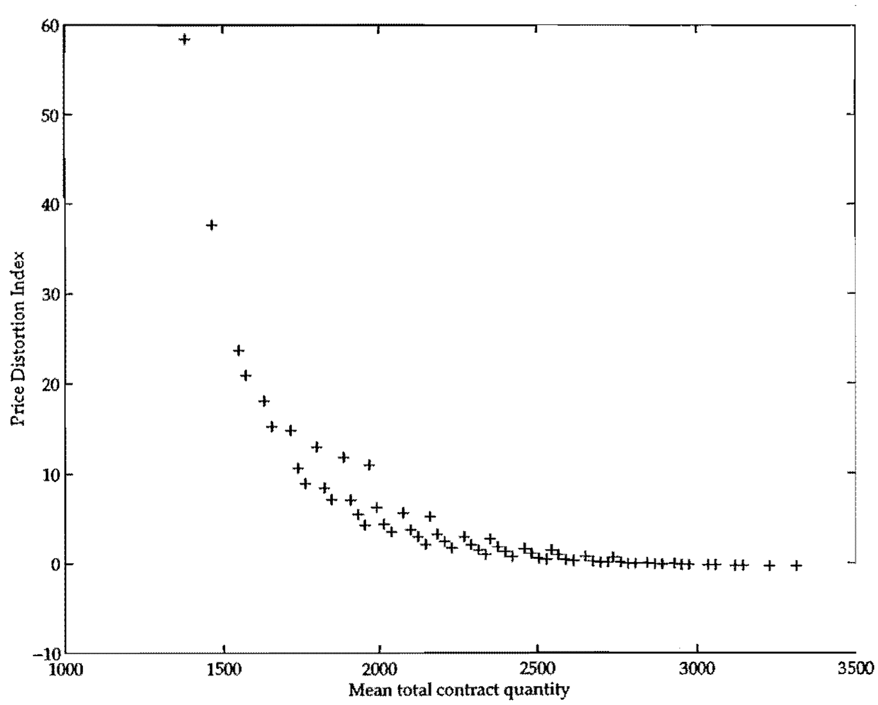

Figure 4.13: Price Distortion Index for a range of contract quantities. No back-up, elasticity of demand -0.1

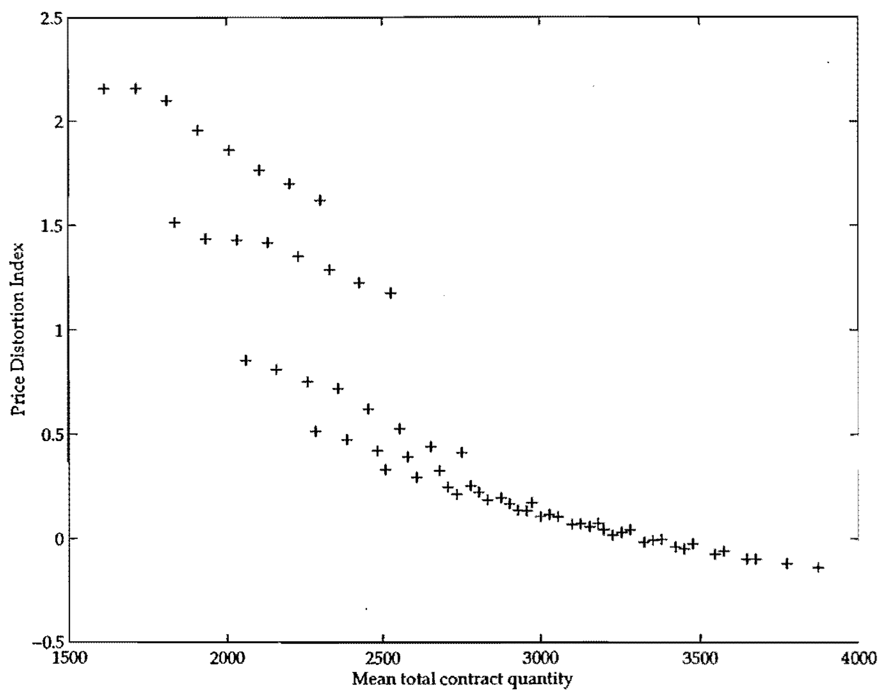

Figure 4.14: Price Distortion Index for a range of contract quantities. No back-up, elasticity of demand -0.3

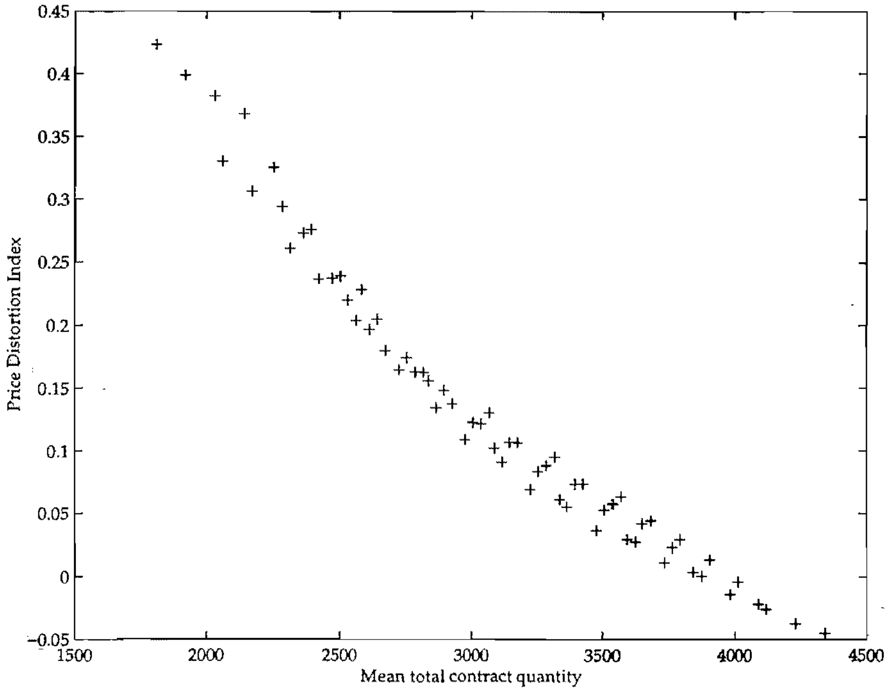

Figure 4.15: Price Distortion Index for a range of contract quantities. No back-up, elasticity of demand -0.8

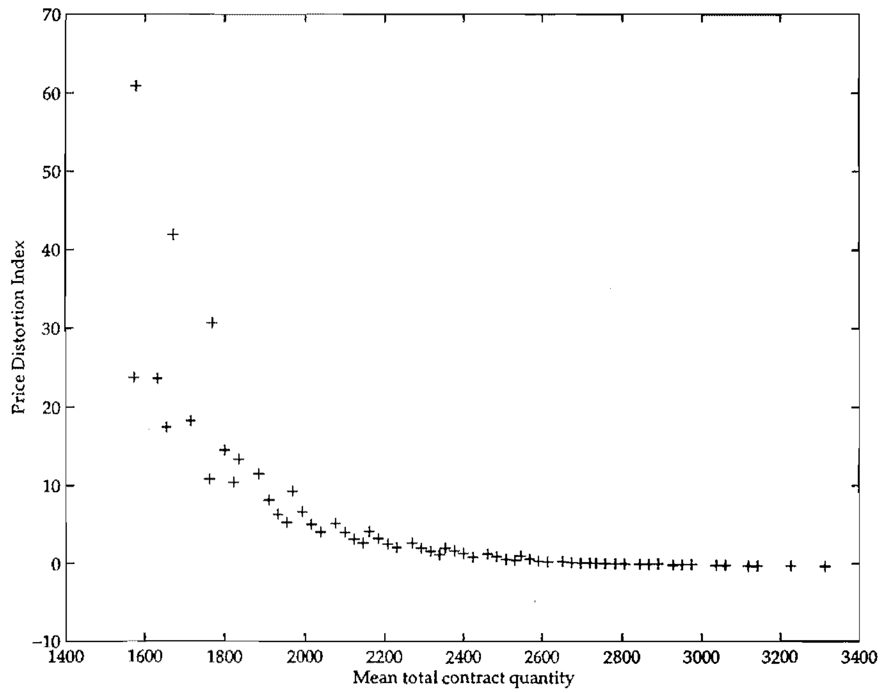

Figure 4.16: Price Distortion Index for a range of contract quantities. 50% back-up, elasticity of demand -0.1

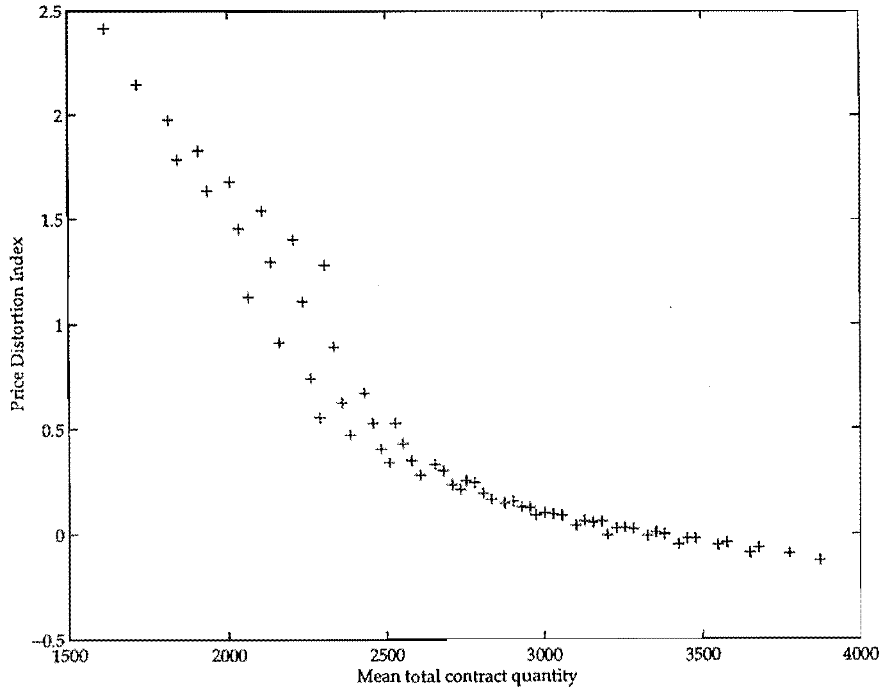

Figure 4.17: Price Distortion Index for a range of contract quantities. 50% back-up, elasticity of demand -0.3

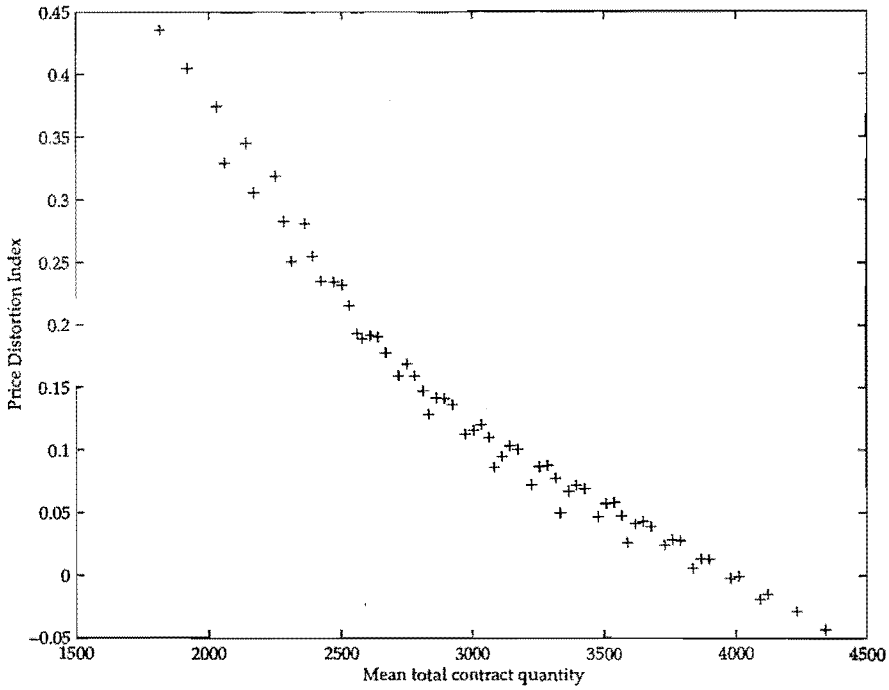

Figure 4.18: Price Distortion Index for a range of contract quantities. 50% back-up, elasticity of demand -0.8

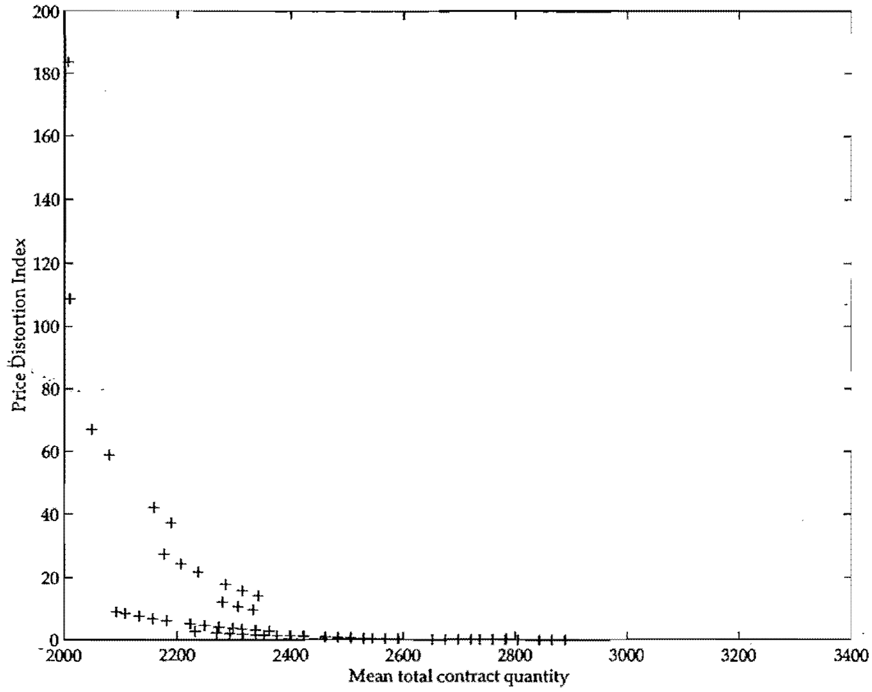

Figure 4.19: Price Distortion Index for a range of contract quantities. 100% back-up, elasticity of demand -0.1

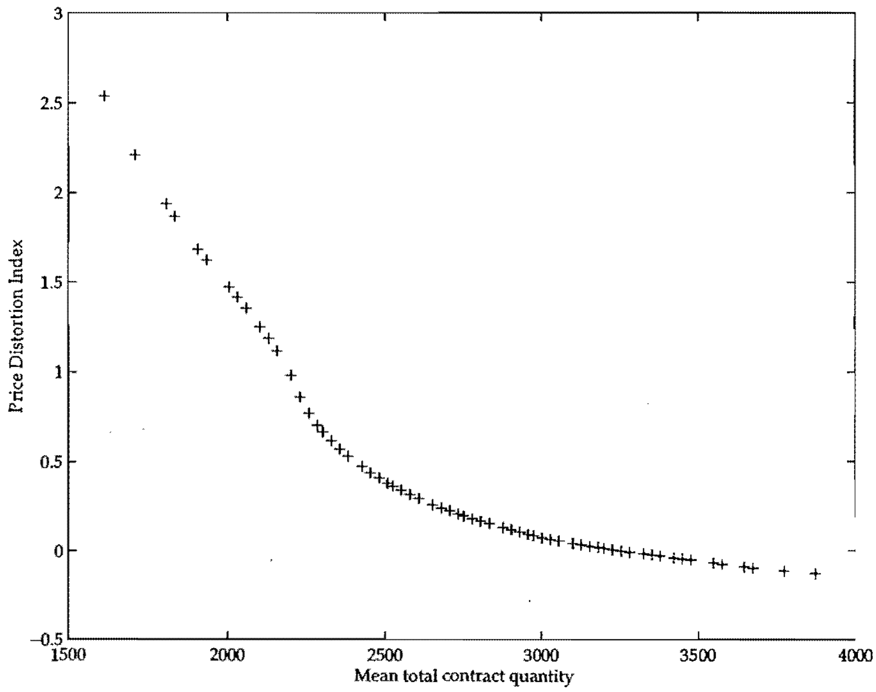

Figure 4.20: Price Distortion Index for a range of contract quantities. 100% back-up, elasticity of demand -0.3

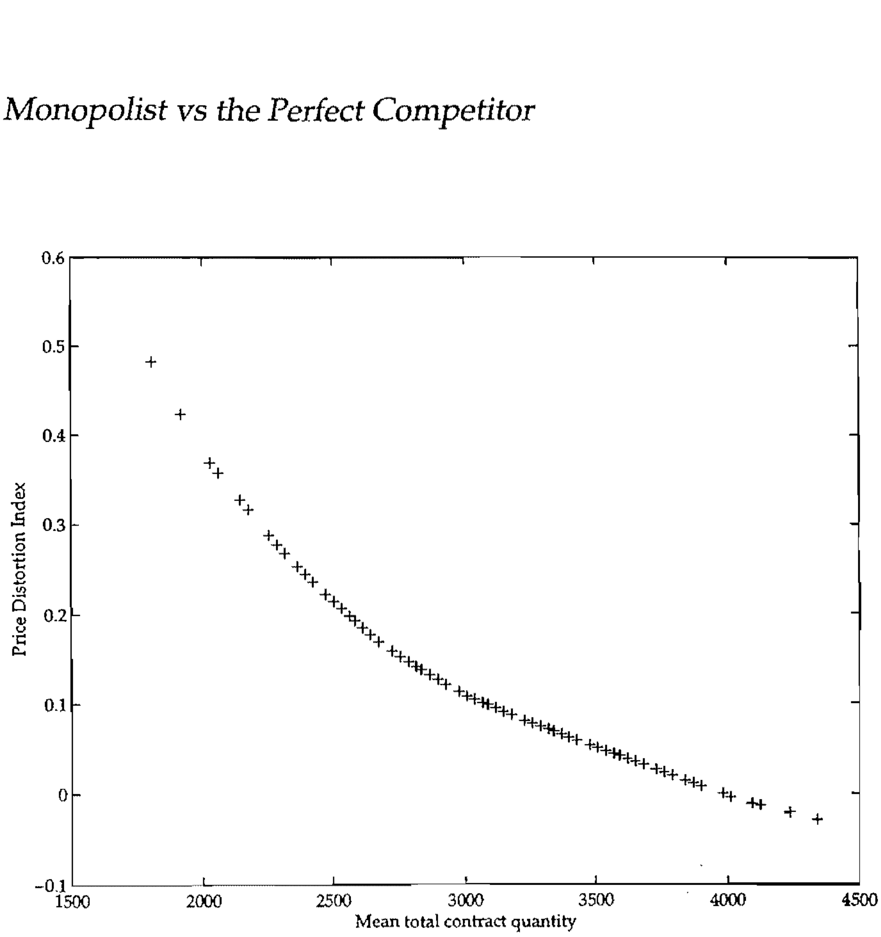

Figure 4.21: Price Distortion Index for a range of contract quantities. 100% back-up, elasticity of demand -0.8

This creates an interesting situation where the monopolist can manipulate the market price so as to influence the PC generation. This will often mean pushing the price up to just below the next highest marginal cost in the Perfect Competitor's merit order. It may also mean that the MRC will be non-monotone, and may even double back on itself, with market output decreasing with increasing price, as illustrated in the extreme case of Figure 4.22. It seems unlikely that an MRC such as that could be put forward to any spot market. The implications of this are that model we present is not entirely plausible under such circumstances. But note that for higher levels of contracting this is much less of a problem.

For more realistic contracting levels of 90% in Figure 4.23 the curve is very nearly monotone, but is still notably different from that in Figure 4.3. In particular note the long flat section where the price is held just below Firm Two's 2.5 cent station, New Plymouth. The individual generation tells the story as we see New Plymouth coming in fully when the price rises to 2.5 cents. Further up the merit order the MRC follows PC more closely than in Figure 4.3, as these are all small stations owned by Firm Two.

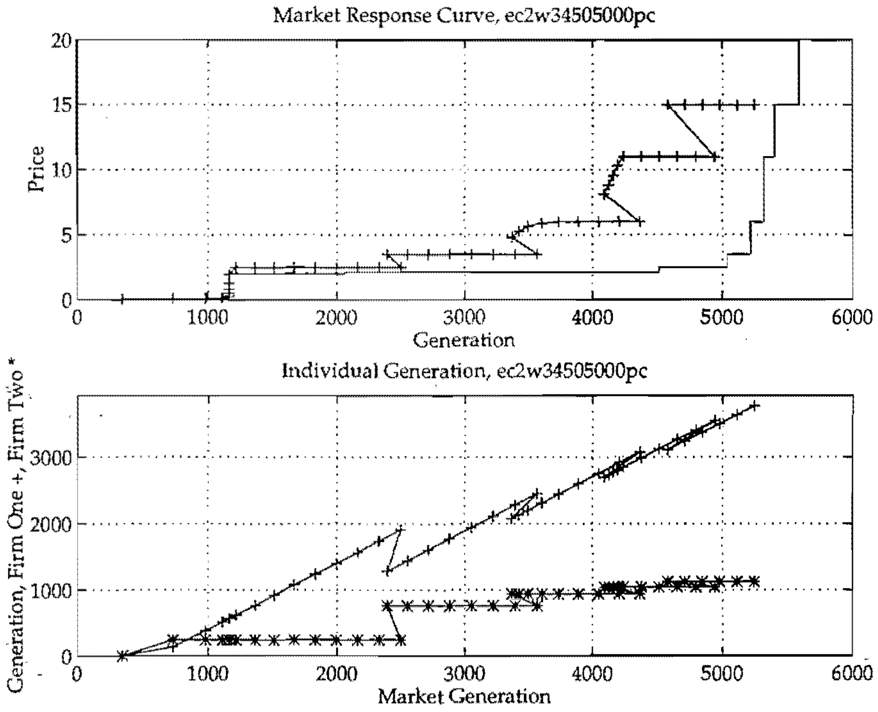

Figure 4.22: Market Response Curve, both Firms 50% contracted, no backup. Firm One is acting as a monopolist, Firm Two as a Perfect Competitor.

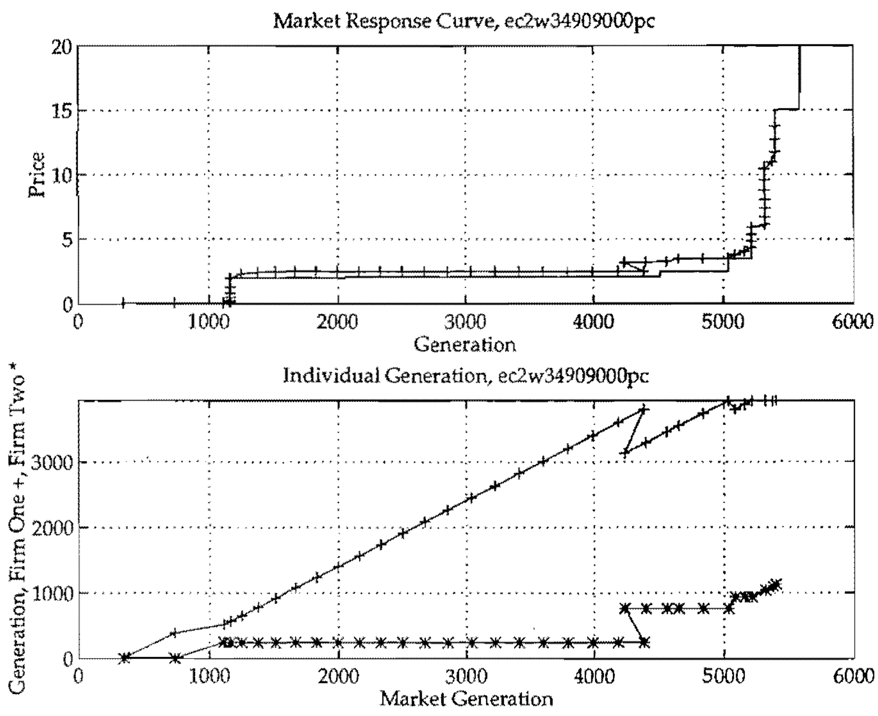

Figure 4.23: Market Response Curve, both Firms 90% contracted, no backup. Firm One is acting as a monopolist, Firm Two as a Perfect Competitor.

As can be seen, the PC has a significant moderating effect, putting a price cap in at each of its marginal costs. The oligopolist cannot push the price any higher than the next highest marginal cost of the PC without the PC coming in with more generation. However the non-monotone MRCs produced may lead to significant problems in modelling, with the possibility of higher demand leading to lower output, but at a higher price. For our purposes, what we will require is that Firm One's demand for water be monotonic-non-decreasing. Fortunately this may still be the case if the non-monotonicities in the MRC occur when the hydro stations are running either fully on or fully off. This is discussed further in Chapter 5.

4.8 Conclusions

We have studied the market response under a range of assumptions including linear and constant elasticity demand curves of various slopes, different market break-up options and different behaviour assumptions.

The level of contracting greatly influences the spot market, with higher levels of contracting leading to lower prices and higher output. In a market with full back-up contracts the price distortion decreases monotonically with increasing market contract quantities. With less than full back-up the price distortion still decreases with each firm's increasing contracts, but the larger firm has a much greater effect than the small firm.

It is not only the total generating capacity, but also the portfolio of generating plant a company holds that influences their market power. Firms with low merit order plant are better able to influence the spot market. Hence letting an under-contracted, low merit order company buy back-up may well increase the price distortions rather than reduce them.

The model is very sensitive to changes in the slope of the demand curves, a point worth noting if quantitative rather than qualitative observations are to be drawn. Steeper (less elastic) demand curves lead to higher price distortions, as is expected. Note that while price is very sensitive, quantities are less so, as can be observed in Chapter 6.

The choice of using linear or constant elasticity demand curves seems on the face of it to be somewhat arbitrary, but as we noted earlier this depends on how far from the reference point the equilibrium is expected to be. Solving for linear demand is simpler as we have a closed form solution where the constant elasticity demand requires numerical solution methods. Perhaps piece-wise linear demand would be a worthwhile compromise to investigate in future studies.

The break-up option *ec2* appears to have greater price distortions than the option *ww*. The distortion seems to be as a result of the imbalance between firms in *ec2*, with Firm One's capacity being nearly 4000 MW, all low merit order, compared with Firm Two's 1300 MW, mostly high merit order.

Finally, although having a PC in the market does have the desirable effect of moderating distortions somewhat, it may also lead to non-monotonicities in the MRC, and hence to troubles in modelling the medium term market behaviour.

Chapter 5

Managing Hydro Reservoirs Over Several Time Periods: Theory

5.1 Introduction

In the previous Chapters we have developed and tested a method for predicting spot prices and generation levels under a given set of conditions. One of the requirements of our short run model (SRM) is that we have the marginal costs of generation for each of the stations, including the hydro station. In order to calculate the marginal water value (marginal cost of generating at the hydro station) we need to consider not only the current time period, but also all future time periods that the water may be used in. In this chapter we detail the procedure we use for calculating the marginal water value. Our method is iterative, and is based on a Dual variant of Dynamic Programming which we call Dual DP (DDP).

5.2 DP and DDP

In the standard (primal) DP approach to reservoir management, ignoring inflow correlation, the stages are time periods (weeks or months perhaps) and the state space is defined by the storage level of the reservoir. The storage level is divided up into some arbitrary grid of storage levels, and perhaps later this grid is refined. At each stage, t, we try to maximise the profit from release during the period plus the value of water left in storage at the end of the period:

$$\text{maximise } v_j^{t-1}(s_j^t) = p^t(g^t)[g_j^t - k_j^t] - c_j^t(g_j^t) + v_j^t(s_j^{t+1}) \tag{5.1}$$

such that

$$s_j^{t+1} = s_j^t + f_j^t - g_{jH}^t \tag{5.2}$$

where $v_j^{t-1}(s_j^t)$ is the value to firm j at the end of period $t-1$ of having s_j^t units of water in storage at the start of period t. The cost of generating g_j^t units is $c_j^t(g_j^t)$, and k_j^t is the contract amount. The cost of generation is, strictly, a function of storage as well as generation, as indicated by the state transformation equation, (5.2). The DP approach is to recursively solve the state equation, (5.1) for all possible values of the state variable, s_j^t (storage). The levels of storage considered are only those on the defined grid. In practice this maximisation problem may well be solved by setting the first derivative of (5.1) to zero, that is, equating marginal revenue with marginal costs. For each storage level there will be an associated price, that being the marginal cost (revenue). For our range of storage levels we will have a corresponding range of marginal values, and an important feature of the optimal solution to our DP is that the storage trajectory will be such that the marginal values are equated from one period to the next[1]. If this were not the case then we could achieve a better solution by carrying more water over from periods were it was of low value to periods where it was of high value. In Dual DP we make use of this feature to turn the problem around.

The DDP approach is dual in that we consider the price rather than the storage level. The state space is defined by the marginal water value (price of water) and this is divided up into some grid. In the PC case this grid is defined at the critical marginal values at which the operating decision will change. These are the marginal costs of the thermal stations, since these determine which stations should be operating. Unfortunately in the market situation we are studying we must consider more than just these critical points as stations are no longer bid in at exactly their marginal costs, and there is a continuum of prices for which the

[1]Marginal water values will be equated except when the reservoir is at one of its storage bounds. At that point the firm's marginal costs are still equated, but they now are made up of the marginal water value plus a multiplier on the storage constraint.

operating decision may change[2].

In DDP we ask at each stage what the required release level would be for a range of marginal water values. In DP the question was what would the release this period and stock carried over be for a range of starting storage levels. The DDP method has an important economic interpretation, which we discuss in the next section.

5.3 Economic interpretation of DDP as applied to reservoir management

In a situation where a supplier can offer the same goods at the same price to several consumer groups, each with known demand curves, the supplier may aggregate the individual demand curves. At any given price, p, say, the supplier can sell d_1 to market 1, d_2 to market 2 and so on up to market n. Hence the total demand they will face if they offer price p will be $D = \sum_{i=1}^{n} d_i$, the sum of all individual consumer demands. This corresponds to adding the individual demand curves along the quantity axis.

The situation faced by a reservoir manager is similar to that of the supplier with many consumer groups. Water in the reservoir this period can be used in this period, or it can be stored for use in the next period. In the next period we will face the same choice of using the water straight away, or storing for later use. Hence our water may be used to generate electricity for sale in any of a number of future spot markets, and the demand for that water will be the aggregate of the demand from all those periods. Note that the number of future periods that need be considered is limited by the storage bounds of our reservoir. If we know the reservoir will be full in some future time period then there is no value in committing any extra water to storage for use beyond that time, as we will not be able to carry it over, and it will be spilt. Similarly our reservoir cannot have a negative amount in storage, so we may never use more water than we have in storage at a given time. These constraints imply that the marginal value of water held in storage is only affected by the value that water would have up until the next time the

[2]While the position of hydro in our firm's merit order won't always change, the quantity we offer to the Cournot market will.

reservoir hits one of its storage bounds.

Continuing with our analogy with demand curves, we will discuss our method in terms of *demand for release* and *demand for storage*. The demand curve for release (DCR) describes the amount of water we would want to release during a given period for a range of marginal water values[3]. The demand curve for storage (DCS) describes the amount of water we would want to hold in storage at the end of a given period for a range of marginal water values. Our method will use backwards recursion, starting with an end of horizon DCS and working back to the beginning of the time horizon.

In DP terms, the DCS represents the solution to the state functions for the range of marginal water values. The DCR is the solution set to the single period objective function for the range of marginal water values[4].

5.4 DDP with general demand curves

The previous implementations of DDP (Read 1985, Read & Boshier 1989, Yang 1995, Macgregor 1991) have all considered the more simple case of a centrally coordinated system, equivalent to PC. In that situation the DCR is piece-wise linear, a simple step function as shown in Figure 5.1, with the steps representing the marginal costs and capacities of the various thermal stations. For a detailed explanation of the procedure see (Read & George 1990), (Macgregor 1991) or (Yang 1995). The computational advantage of having a stepped DCR is enormous. The procedure of adding the DCR to the DCS is replaced by one of *inserting flats* into the DCS. Indeed it is this fact which allows (Macgregor 1991) to report that the number of additions required for a 12 period stochastic DP with 5 steps in the

[3]If we separate the Hydro Station Manager's problem into two problems, one of storing water for sale and the other of buying water to generate with, then the marginal water value is the price at which water will be traded between the two problems. The DCR describes the amount to be released within the period for a range of marginal water values, and the DCS describes the amount to be stored for future use.

[4]On the face of it it would seem that our method still uses storage as the state variable in terms of the state transformation. In fact we believe that the state transformation function is no longer the conservation of flow, but now the equating of marginal values over time. The reason for the confusion is that our DCR represents the solution for the full range of the state variable, and contains both the primal and the dual information. Thus is it difficult to separate the two in our framework.

marginal cost function was over 220000, yet for the same problem solved via stochastic DDP the number of additions required was around 5000. The solution time for SDDP was less than 2% of that for SDP.

The case of a stepped DCR is just a special case of our requirement that the DCR have the same properties that are normally expected of a demand curve: it must be continuous and the slope must be everywhere non-positive.

When modelling a competitive market the DCR is no longer a step function, and the computational advantage of DDP may be largely lost. However the concept is still valid, and intuitive. Further, when viewed as a process of adding demand curves it does not require that there be a well defined underlying benefit function to optimise[5].

5.5 Stepping Back One Period

As is typical in dynamic programming, we start with the end of the time horizon and work back recursively to build up our description of the solution strategy for all periods. We start with an end of horizon DCS and derive the DCS for the previous periods from it. Reference to Figure 5.2 may help in the understanding of this process. At the beginning of the final period, each additional unit of water we hold in storage could either be used during the period or carried over to the end of horizon at known marginal value, as defined by the end of horizon DCS. At a particular marginal water value, ψ say, we may expect to sell r_t units within the period as defined by the DCR, and s_{t+1} units at the end of the period, as defined by the DCS. Adding these together gives a total demand for water at the price ψ of $r_t + s_{t+1}$. We proceed similarly for all values of ψ to get the nett demand for water as the sum of the two individual demands, that within the period (the DCR) and that for the end of the period (the DCS)[6]. This corresponds, as mentioned earlier, to adding individual demand curves to give a total aggregate demand for the whole market. We have not yet taken into account the expected inflow within the

[5]Integration over the DCR would produce a well defined benefit function, to which we could apply conventional DP. It is not clear exactly what this benefit function represents, but it is certainly not the overall benefit to the firm. It seems likely that it corresponds to the hydro managers subproblem, but we leave this as a topic for further research.

[6]Note that we do not consider discounting, wastage, head effects etc., but see Macgregor (1991).

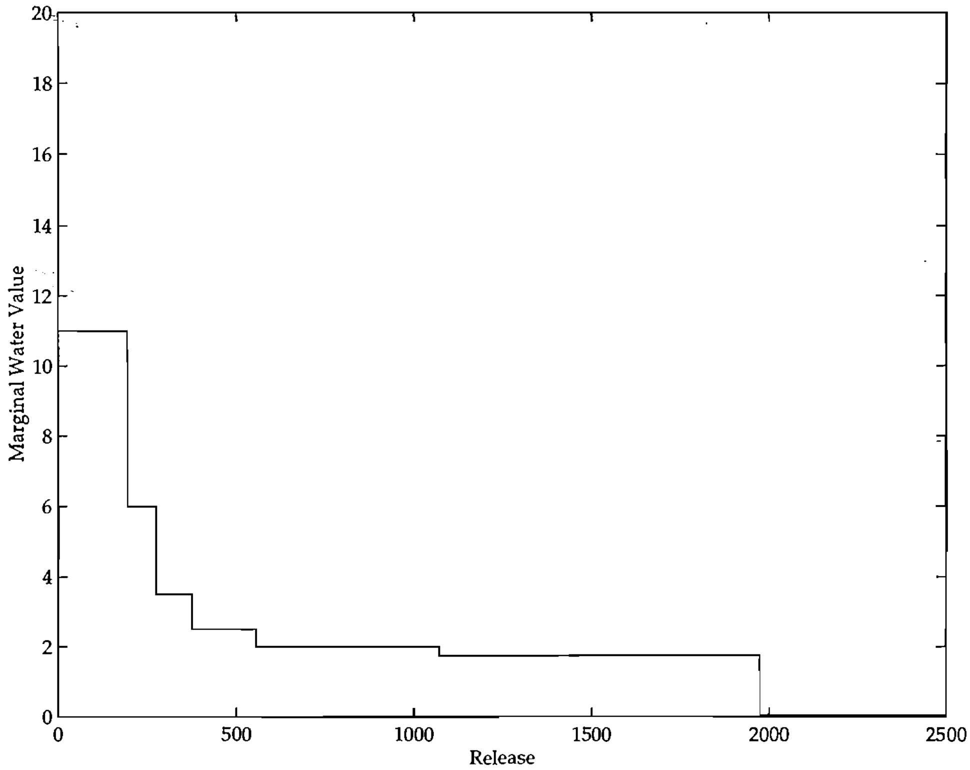

Figure 5.1: A demand Curve for Release (DCR) for a centrally coordinated system. This DCR is a piece-wise linear stepped curve, which would enable us to make considerable computational gains in our DDP algorithm. Unfortunately it is not typical of the DCRs we get from our Cournot Model.

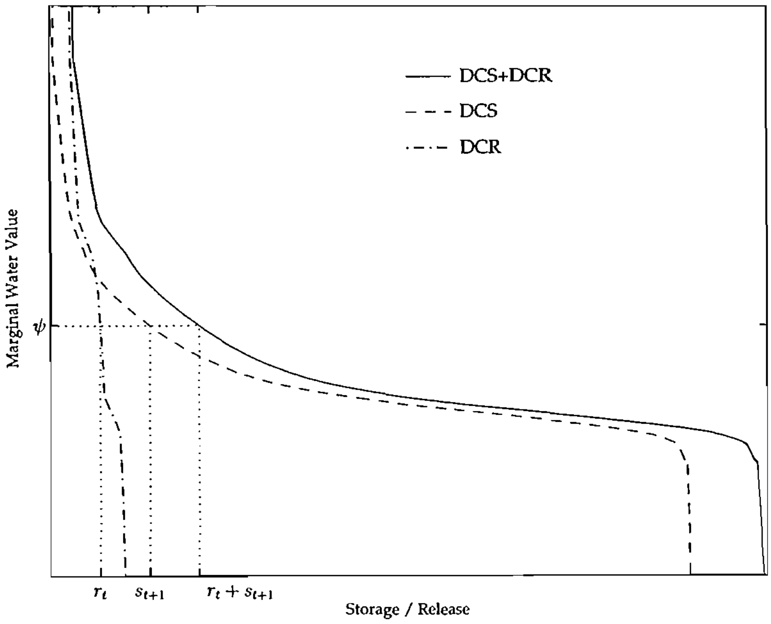

Figure 5.2: Adding the end of period DCS to the within period DCR.

period, and we do this in the next section.

5.5.1 Adding Inflows

The aggregate demand curve we have tells us the value of each additional unit of water we have in storage at the beginning of the period. We assume that all the inflow, F, arrives at the end of the current period, after the decisions for this period have been made. There is no cost for receiving the inflow, and hence the value of holding s units of water in storage at the beginning of this period is equal to the value of holding $s + F$ units in storage at the end of this period. Hence we simply shift the end of period curve to the left by the inflow amount to get the beginning of period curve, which is directly equivalent to the end of period curve for the previous period[7].

In practice we are not likely to know exactly what the inflow will be in a given period, especially many months ahead. We have historical data though, which

[7]There are no release decisions made, nor any inflow received, during the instant between the end of one period and the beginning of the next, so values at the end of one period are exactly the same as at the beginning of the next period.

we could use to estimate a range of likely inflows, and their likelihood of occurring. This distribution provides us with a set of possible end of previous period marginal value curves, each with a given likelihood.

Let us assume that the inflow, F, is a discrete random variable[8] with values drawn from the set of possible inflows, with associated probabilities μ:

$$F_t \in (f_{1t}, \cdots, f_{nt}), \tag{5.3}$$

$$\mu_{it} = \mu(f_{it}) = P[F_t = f_{it}], \tag{5.4}$$

and the expected value is

$$E(F_t) = \sum_{i=1}^{n} \mu_{it} f_{it} \tag{5.5}$$

For any given beginning of period storage level for the current period s_t, say, with associated marginal water value ψ_{st}, the expected storage level for the beginning of the next period, after inflows, will be

$$s_{t+1} = s_t + E(F_t). \tag{5.6}$$

However in DDP our main concern is the expected marginal water value. For a given storage level, s_t, the marginal water value is $\psi_{t+1}(s_t + f_{1t})$ if inflow f_{1t} is going to occur and $\psi_{t+1}(s_t + f_{2t})$ if inflow f_{2t} is going to occur. The expected marginal water value is then

$$E(\psi(s_t)) = \sum_{i=1}^{n} \mu_i \psi_{t+1}(s_t + f_{it}). \tag{5.7}$$

This is illustrated in Figure 5.3 for the simple case of two equally likely inflows, and in Figure 5.4 for five inflow scenarios with weights of $\begin{bmatrix} 0.15 & 0.20 & 0.30 & 0.20 & 0.15 \end{bmatrix}$. In Figure 5.3 the curve labelled *End of period DCS + DCR* corresponds to the solid curve in Figure 5.2. It represents the marginal value of water in storage at the end of period t, just after the inflow arrives (and is known). The two dashed curves represent the marginal value of water in storage just before the inflow arrives, assuming the inflow will be f_{1t} and f_{2t}, respectively. For a particular beginning

[8] We assume there is no correlation from one period to the next, but see (Yang 1995) for a treatment of correlation

of period storage level, s_t say, (before the inflow arrives) we expect the marginal value of water in storage to be $\psi_{t+1}(s_t + f_{1t})$ if inflow f_{1t} occurs, and $\psi_{t+1}(s_t + f_{2t})$ if inflow f_{2t} occurs. Hence the expected marginal value of water in storage is the probability weighted average

$$E(\psi_t) = \mu_{1t}\psi_{t+1}(s_t + f_{1t}) + \mu_{2t}\psi_{t+1}(s_t + f_{2t}). \tag{5.8}$$

This leads to the beginning of period expected DCS, which becomes our end of period DCS for the previous period at the next iteration.

What of the parts of the new DCS for which storage lies outside the bounds? If water spilt has no value then there is no value in holding extra water in storage now that will need to be spilt next period, so storage levels above the upper bound are assigned a marginal water value of zero. Similarly if we have a fixed shortage cost, as is the case with our model[9], then the marginal value of any water used that will bring the storage level back to the lower bound is just this shortage cost, as each extra unit we carry over will be used to curtail a unit of shortage[10]. The parts of the new DCS that are above the upper storage bound or below the lower storage bound are hence truncated at the marginal value of water spilt or of shortage, respectively.

Our complete process for deriving the DCS for the end of a period given the DCS for the end of the next period, the DCR for within the next period and the set of possible inflows is as follows.

1. add DCS for next month to the DCR for next month (along storage axis)

2. subtract inflow for each possible inflow level, f_i, to give the set of possible DCS

[9] A fixed shortage cost could also be thought of as a price cap on the short term energy spot market. This might be an actual cap imposed by the rules of the NZEM, or a perceived cap, imposed by threat of regulation, or threat of entry. It might also be a cap implied by a one way contract limiting liability to a certain marginal cost.

[10] Although this is true for the centrally coordinated system, it is not strictly correct for the market situation. Consider the example of a small firm whose reservoir is empty, and whose thermal capacity is fully utilised. Extra units will come not at some arbitrary shortage cost, but at the energy spot price, which may be lower or higher than the value we have imposed in our model. However tests by Yang (1995) indicate that this effect is relatively minor for a single national reservoir, and since it is not directly relevant to the concerns of this thesis we ignore it here.

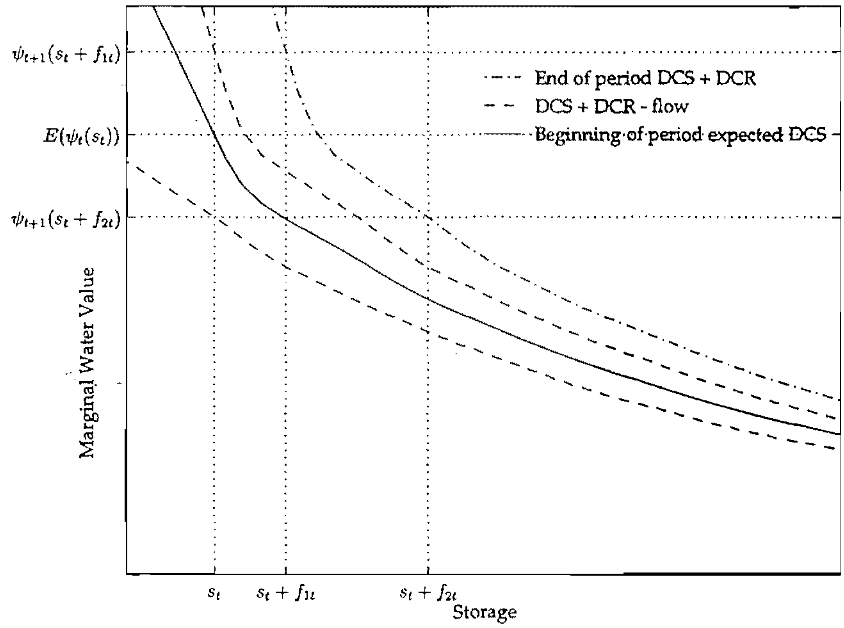

Figure 5.3: Accounting for uncertainty in inflows; two equally likely inflows.

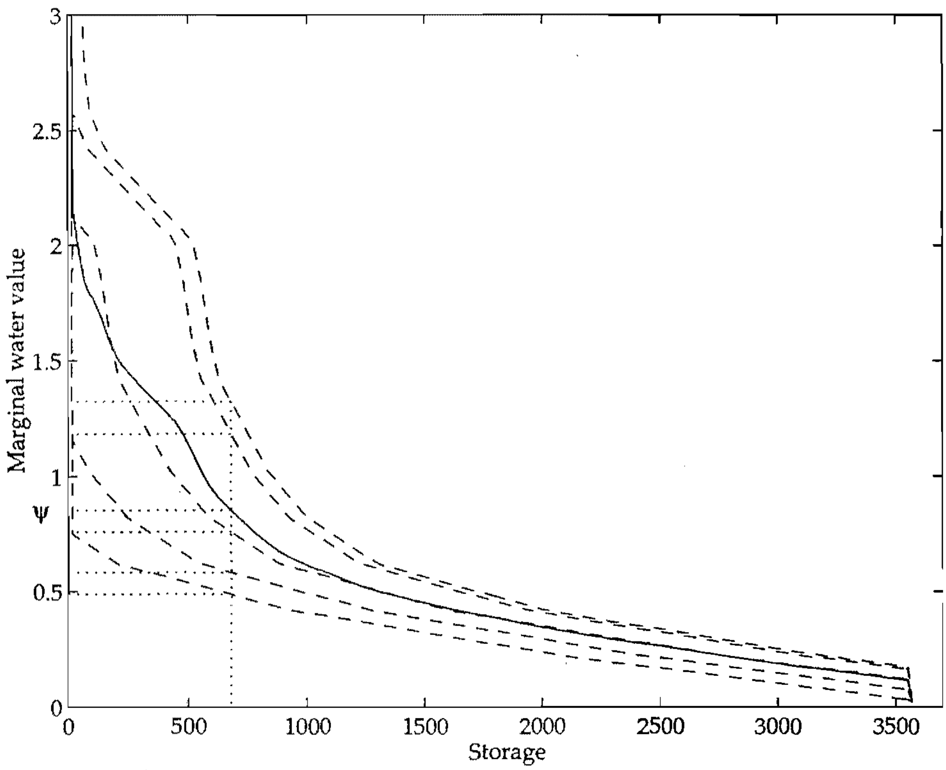

Figure 5.4: Accounting for uncertainty in inflows.

3. average the DCS along the marginal water value axis according to the probability weights μ_i

This process is repeated, starting at the end of horizon, stepping back through each period to the beginning of horizon. At this stage we have what we term a *water value surface* (WVS) such as that shown in Figure 5.5.

The only part we have not explained is how we derive the end of horizon DCS. In practice this may be given from outside of the model, for example as an end of year accounting policy. For our purposes we simply extended the model far enough out into the future that the end conditions were not important. To test for this, we compared the DCS for the end of horizon to the DCS for the same period one year earlier. In equilibrium, given our annual inflow and demand patterns, these two DCSs should be the same. If they are not, we simply continue our DDP process for another year, and check again. In practice the process seems to converge between two and three years into the future[11].

5.6 Operating Rules and the Water Value Surface

The WVS we have constructed using the procedure detailed in the previous sections can provide a wealth of information with regards to the operation of our hydro and thermal stations.

In any given period we may observe the storage level of our reservoir, and then read the implied expected marginal water value directly from the water value surface. Once we know the marginal water value we can schedule our system like a pure thermal system, with the MWV being the fuel cost of hydro. One method for doing this is that explained in Chapter 3.

The level contours of the WVS are often referred to as *guidelines* because they indicate where hydro fits in our company's merit order. If the contours are drawn for MWVs equal to each of our own marginal thermal costs then the storage-time region is divided into bands. For each thermal station that has a marginal cost lower than the MWV the thermal station should be fully utilised before we start using the hydro. For each thermal station that has a marginal cost higher than the

[11]Some DDP iterations could be saved by checking at each week to see if we have achieved convergence, rather than just at the end of the horizon, but we did not implement this strategy.

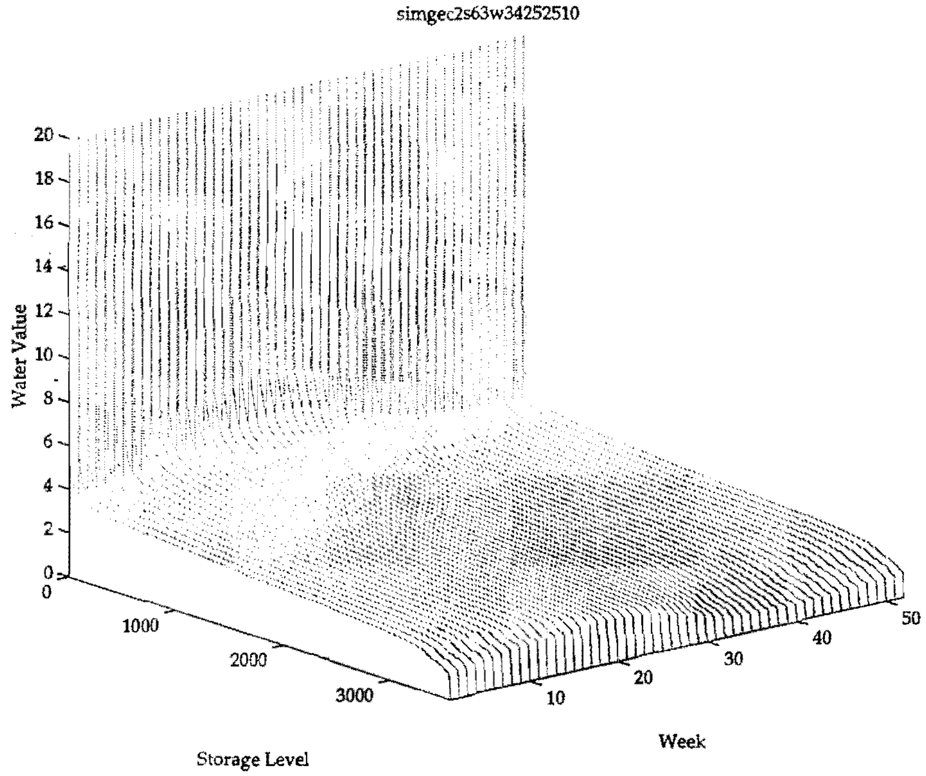

Figure 5.5: A typical water value surface.

MWV the thermal station should not be used at all until hydro is fully utilised[12]. These level contours, then, provide a means of correctly placing hydro in our merit order.

5.7 Demand Curves for Release

As mentioned throughout this chapter, our DDP method requires us to calculate DCRs for each period throughout the time horizon. Given a method for predicting the spot market outcome for a set of input parameters, the process of calculating the DCRs is relatively simple.

DDP requires that the DCR be a (non-strictly) monotone decreasing function, as is typically expected of demand curves. Three example DCRs which meet this requirement are shown in Figure 5.6. Two which don't are shown in Figure 5.7. As plotted the DCRs suggest that release is the independent variable, and price the dependent variable. Our method for deriving these curves takes the reverse approach.

Remember that our SRM will tell us the spot price and generation levels for a given set of marginal costs and other input information. To derive the DCRs we run the SRM with a representative range of MWVs, holding all other inputs constant. For each MWV we record the corresponding hydro generation. This we now think of as the demand for hydro generation for that given price (MWV). Plotting hydro generation vs. price for the range of MWVs gives us our required DCR.

Does the DCR we generate in this fashion meet our requirements for DDP, that it be a decreasing function?

Theorem 5.1. *For the maximisation problems described in Theorems 3.2 and 3.4 the DCR derived by the method described above is a decreasing function.*

Proof. The proof follows from the proof of Theorem 3.2, where we noted that an increase in marginal cost, or in this case marginal water value, implies a decrease in the equilibrium generation for our firm. If the internal merit order is unchanged

[12] In practice exceptions are made for the purpose of meeting reserve requirements, which may lead to out of merit order operation. However if spinning reserve is correctly priced (Drayton-Bright 1997) then the effective marginal thermal costs will include a spinning reserve component, and merit order is preserved. Reserve is not considered here.

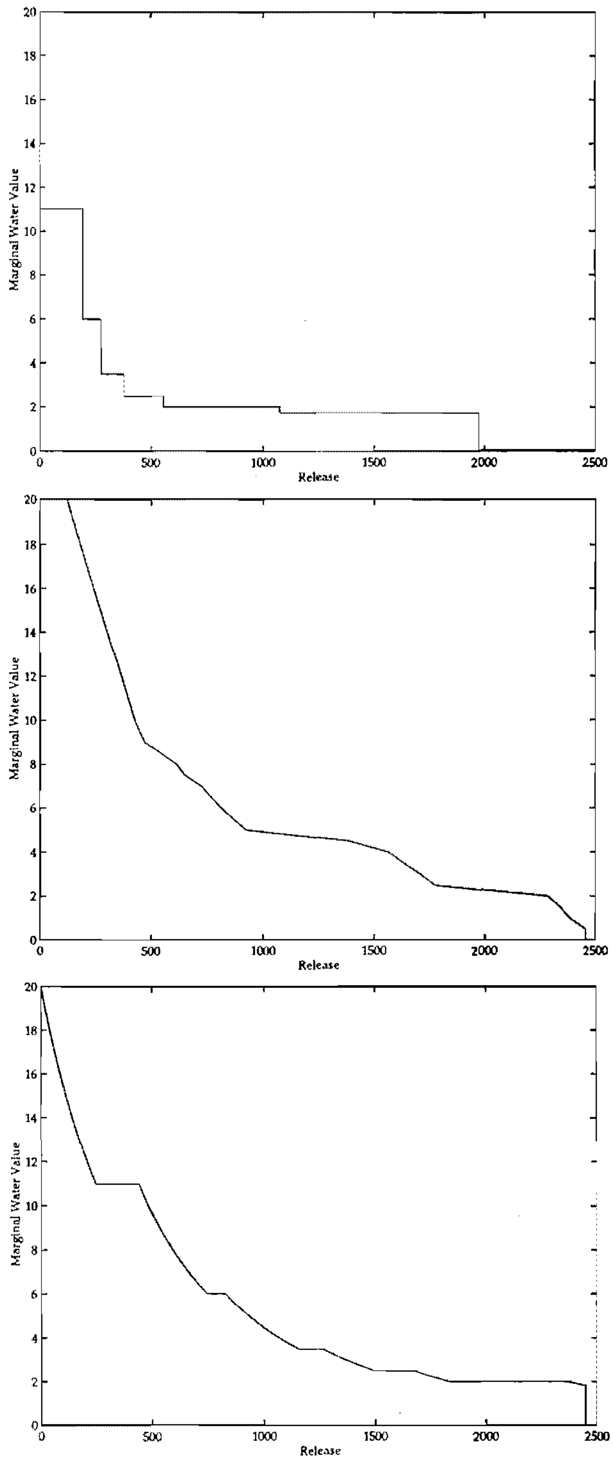

Figure 5.6: Three possible DCRs.

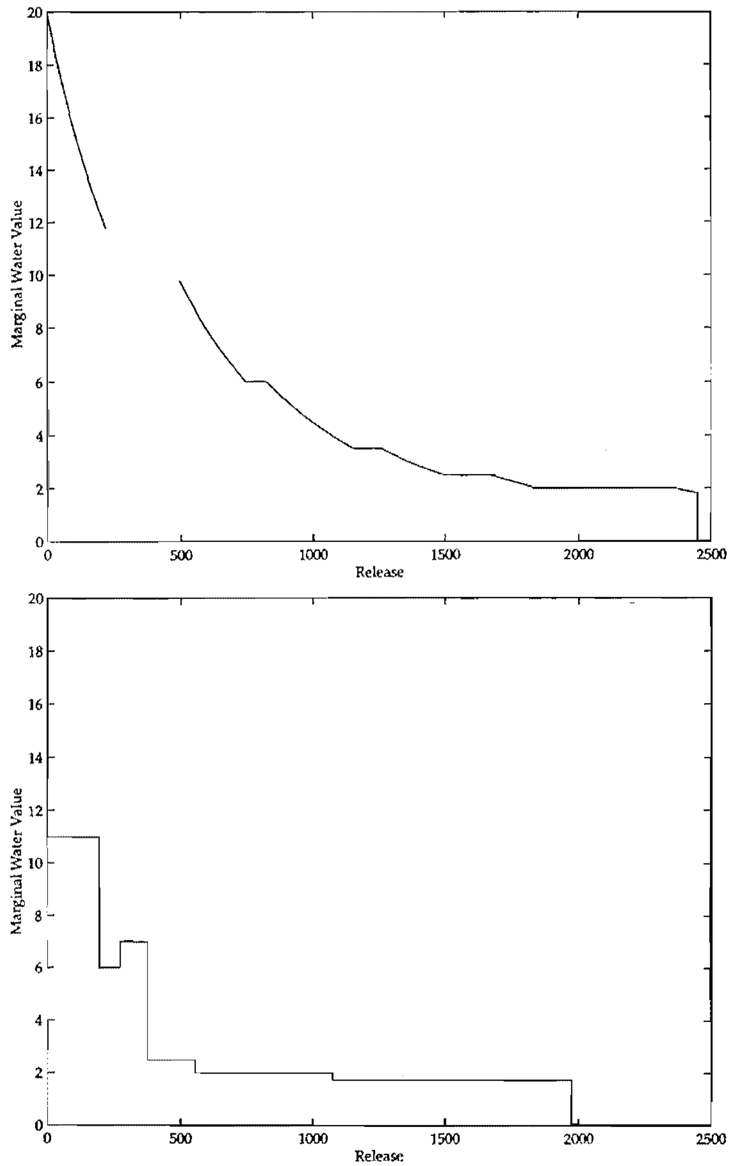

Figure 5.7: Two unacceptable DCRs.

the increase in marginal water value must imply that either the hydro output is unchanged and the thermal output will decrease, or that the hydro output will decrease. If the merit order does change then hydro must move to a more expensive place in the order, and will be less utilised. □

It should be noted that there is a clear distinction between the DCRs and the MRCs of Chapter 4. While the DCRs are calculated for a particular demand/contract scenario and a range of MWVs, the MRCs are for a particular set of marginal costs and for a range of demand scenarios and corresponding contracts. While under reasonable conditions (see § 3.9.1)we can expect the DCR to be monotone, the same cannot really be said about the MRC, especially if one of the firms is a price taker, and the other firm can exploit this. In such cases it is quite possible that a higher level of demand will be met with reduced output. An example of this is shown in Figure 4.22.

In practice we model sub-periods with an LDC, representing the range of demands likely within the period. This is no problem for our method, and the monotonicity of the curve is still maintained. The DCRs for each of the sub-periods will be monotonic, as shown earlier. We combine them by taking a simple weighted average, that is a convex combination, of the sub-period DCRs, and the convex combination of a set of monotone functions is a monotone function (Lasdon 1970).

As well as requiring the DCRs for each period to be monotone, our DDP approach requires the DCS in each period to be monotone. Theorem 5.2 considers this.

Theorem 5.2. *The DCS produced by the addition of a continuous monotone decreasing DCR to a continuous monotone decreasing DCS, as detailed in § 5.5, will also be continuous monotone decreasing. Further, so will the augmented DCS after accounting for the inflow adjustment, including uncertainty.*

Proof. By assumption the DCS and the DCR are continuous monotone decreasing, and hence invertable. We can, then, consider the range of marginal water values from zero through to the maximum for which either the inverse DCR (DCR^{-1}) or the inverse DCS (DCS^{-1}) has a positive value. Both DCR^{-1} and DCS^{-1} will be defined for all these values[13], and so we may add their values to produce the

[13]For simplicity we assume the inverse functions are set to zero for all values where the marginal

new inverse DCS, DCS^{-1}. As we increase the marginal water value, the value of DCS^{-1} and DCR^{-1} both decrease, and so, therefore, does the new function. Our new function is defined over the continuous range of water values, and is monotonic decreasing, and hence invertable to produce the new DCS.

The inflow adjustment can be thought of as shifting the new DCS back along the storage dimension by the (stochastic) inflow amount. Translation will certainly preserve our desired properties of being continuous monotone decreasing. As explained in § 5.5 the uncertainty adjustment for the inflow can be thought of as taking the weighted average of a set of DCSs, shifted back by the different possible inflow amounts (see Figure 5.3). We again have the situation where we are taking the convex combination of a set of monotone functions, and our resultant DCS will once again be a continuous monotone decreasing function. □

We have now ascertained that each DCR corresponds to the optimal release decisions for the corresponding stage of the DP (5.1)–(5.2). By Theorem 5.1 we know that the DCRs are monotone decreasing, and by Theorem 5.2 we also know that the DCSs will be continuos monotone decreasing. These DCSs now fully describe the optimal release policy for any starting marginal water value (or storage). That the policy is optimal we know from our requirement that marginal value be equated across the stages. This has been achieved by our method of adding demand curves.

As mentioned earlier, the set of DCSs combined give us a marginal water value surface which allows us to determine the marginal value of water in any period for any storage level. Once we have this marginal water value we may use an appropriate single period model, such as that described in Chapter 3, to determine optimal release levels, which is the ultimate goal of the reservoir management problem.

5.8 Observations

We have presented our DDP approach to the standard reservoir management problem, but with the added feature that the single period objective function is now

water value exceeds the range of the original function.

described by a market model, rather than a cost minimisation model, as had been done in the past. Our method has a strong analogy with the economic concept of adding consumer demand curves to get an aggregate demand curve. In our case this aggregate demand curve (the DCS) describes demand for water for all future periods, at each of a range of marginal water values (prices).

This method can be used in the absence of a well defined benefit function, and requires only that we can calculate release (demand for water within the period) as a function of the marginal value of water. One method for doing this is via the Cournot game we described in Chapter 3.

The WVS shown in Figure 5.5 has a large relatively flat section. The reason for this flat section can be seen if we return to the concepts described earlier in this Chapter. It is a result of two features. The first is that the DCRs for many of the periods had a relatively flat section around that marginal water value, due to the large thermal station owned by the hydro firm. The second is due to the effect of repeated addition of DCRs and subtraction of inflows. This results in a shear about the marginal water value (of release) corresponding to the mean inflow level.

The steps of each iteration of the DDP process are to add the DCS to the DCR and subtract off the inflows. In the deterministic case, where we know the inflow, the results of the addition of the DCR and the subtraction of inflows cancel each other exactly at the marginal value corresponding to the inflow level, f, that is $\psi_R(f)$. For the stochastic case this applies to each of the shifted DCSs at their respective inflow levels, and hence to the expected DCS (the weighted average of the possible DCSs) at the marginal water value which is the weighted average of the marginal values corresponding to each of the possible flows. This is illustrated in Figure 5.8 for the simple case of two possible inflows, f_1 and f_2. The marginal values of releasing f_1 and f_2 are $\psi_R(f_1)$ and $\psi_R(f_2)$, respectively. The expected marginal value of holding $\bar{f}$ in storage at the beginning of the period is $\psi_S(\bar{f})$. As can be seen the nett effect is to shear the DCS about the point $(\bar{f}, \psi(\bar{f}))$, leading to a flatter and flatter section at each iteration. Of course the expected inflow changes from period to period, so the WVS is not completely flat.

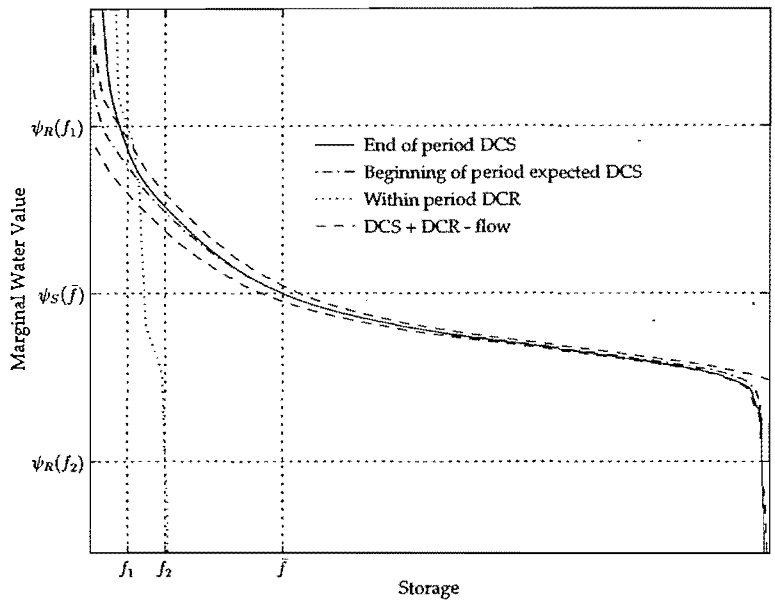

Figure 5.8: Example of the DCS pivoting about the marginal value of the expected inflow level.

Chapter 6

Multiple Period Results

6.1 Introduction

While the results and observations regarding the spot market are useful and interesting in their own right, our real goal is to consider the possibilities in the medium term, especially regarding the effects of competition and contracts on reservoir management. We finally address these issues in this chapter. The chapter is arranged as follows:

§ **6.2** A description of the system parameters and a presentation of the results for the PC case.

§ **6.3** The process of calibrating the optimisation.

§ **6.4** Results for the *ec2* model.

§ **6.5** Results for the *ww* model.

§ **6.6** Other results of interest.

§ **6.7** Conclusions

6.2 The base case

We have built a model which represents most of the important features of the New Zealand electricity sector with respect to reservoir management in the medium term. Our model is of a duopoly with an inactive fringe[1]. We only consider reservoir management for one of the duopolists, and only for one aggregate reservoir. We do not model any transmission constraints, losses or reserve requirements. Nor do we consider the issue of correlation, either of inflows or of demand.

As detailed earlier in this thesis we model the spot market as a Cournot duopoly with a competitive fringe, and we use this to build up reservoir release rules for our medium term simulation. In these experiments the simulation runs for twenty simulated years producing data on generation, release, spot price, consumer and producer surplus, storage and fuel costs.

In this chapter we compile these data for a wide range of conditions and compare and contrast them with the PC case. We find, as we expect given our earlier results, that our model is very sensitive to the level of contracting and the elasticity of demand.

The results presented in this chapter are in either of two forms. The first form is graphical and represents a summary of the means and standard deviations for a set of simulations, with each point plotted coming from a different contract allocation. The second form is also graphical, and in most cases this is a plot of the distribution of results for the twenty simulated years. For the graphs lines are drawn at the 5, 20, 50, 80 and 95% levels. As an example, for the 20% line, in any given week 20% of the time the measured value from the simulation was at or below that level.

The station capacities are as described in Table 6.1 for winter and Table 6.2 for summer. Note the extra capacity of Clutha in the summer, which leads to higher output for Clutha in summer than in winter, as depicted in Figure 6.4.

The reference level we make comparisons against is the PC case. Figures 6.1–6.11 present the PC results for generation, price, water value, storage level, profit and consumer surplus.

[1]The fringe is modelled as having a constant level of output for each sub-period. The output does vary from one sub-period to the next. In an alternative model which we have developed there is one Cournot oligopolist playing against a fringe of perfect competitors, each bidding in at marginal cost. Simulation results from the alternative model are not presented in this thesis.

Station(s)	Capacity	Marginal Cost	Owner
Waitaki and Waikato[2]	2455	2.0	Firm 1
Huntly	900	2.0	Firm 1
New Plymouth	518	2.5	Firm 2
Stratford	178	3.5	Firm 2
Marsden A	103	6.0	Firm 2
Otahuhu	81	11.0	Firm 2
Whirinaki	194	15.0	Firm 2
Clutha	244	0.0	Firm 2
Manapouri	570	0.0	Firm 1

Table 6.1: Break-up option *ec2*, winter levels.

Station(s)	Capacity	Marginal Cost	Owner
Waitaki and Waikato[3]	2455	2.0	Firm 1
Huntly	900	2.0	Firm 1
New Plymouth	518	2.5	Firm 2
Stratford	178	3.5	Firm 2
Marsden A	103	6.0	Firm 2
Otahuhu	81	11.0	Firm 2
Whirinaki	194	15.0	Firm 2
Clutha	639	0.0	Firm 2
Manapouri	570	0.0	Firm 1

Table 6.2: Break-up option *ec2*, summer levels.

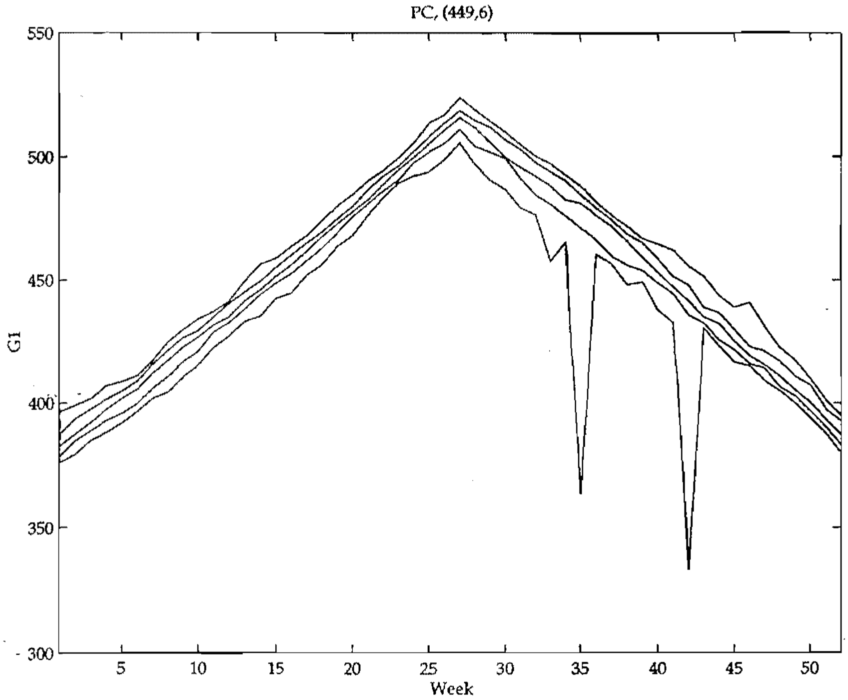

Figure 6.1: PC total generation, Firm One.

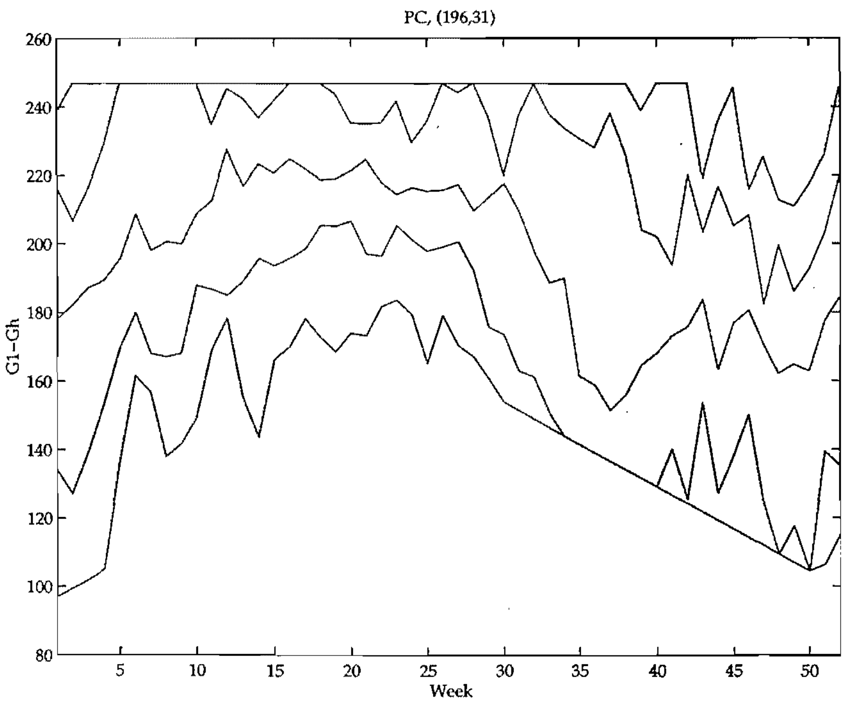

Figure 6.2: PC thermal generation, Firm One.

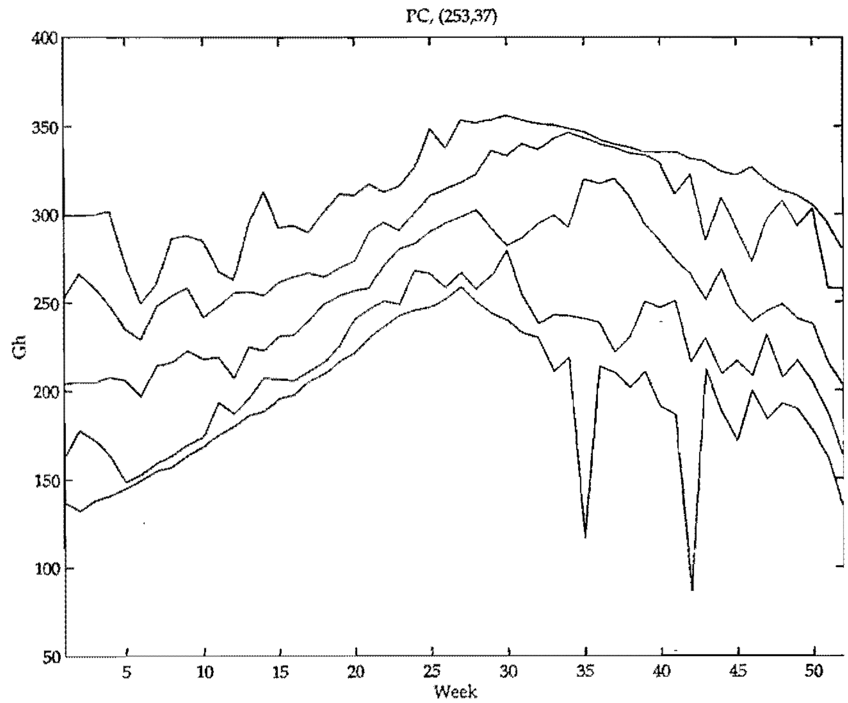

Figure 6.3: PC hydro generation.

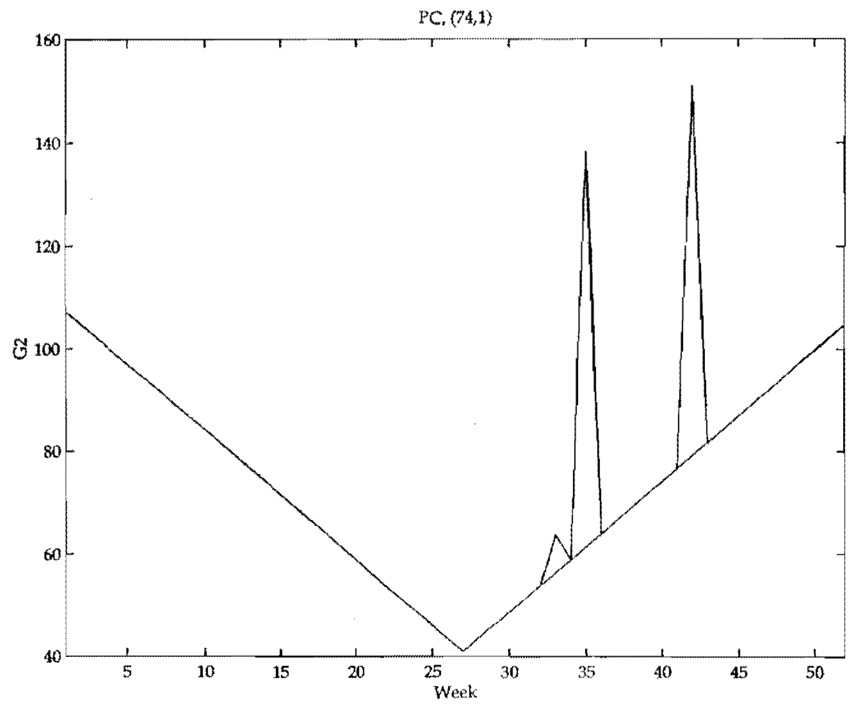

Figure 6.4: PC generation, Firm Two.

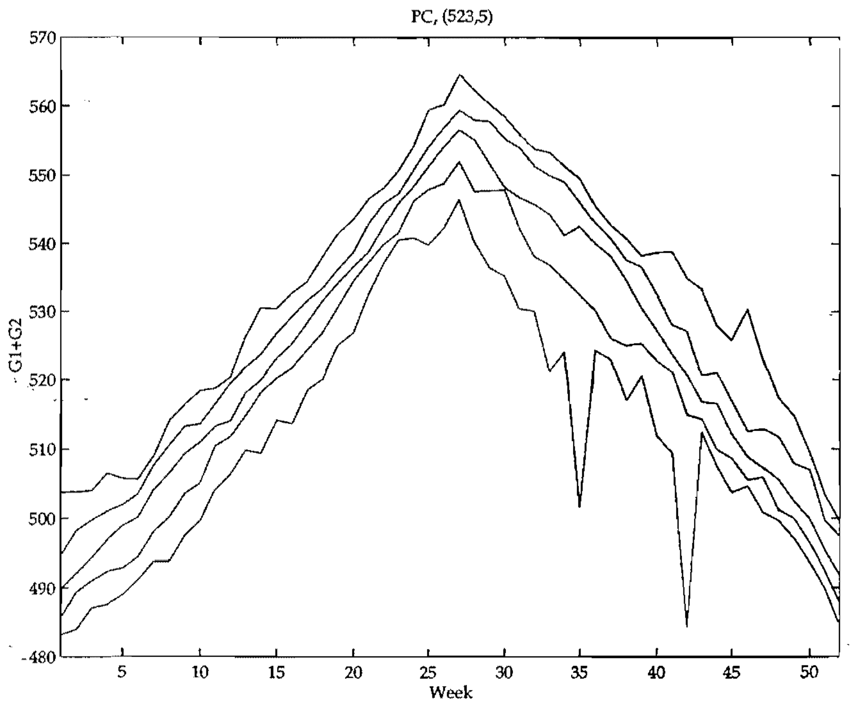

Figure 6.5: PC total generation, Firm One plus Firm Two.

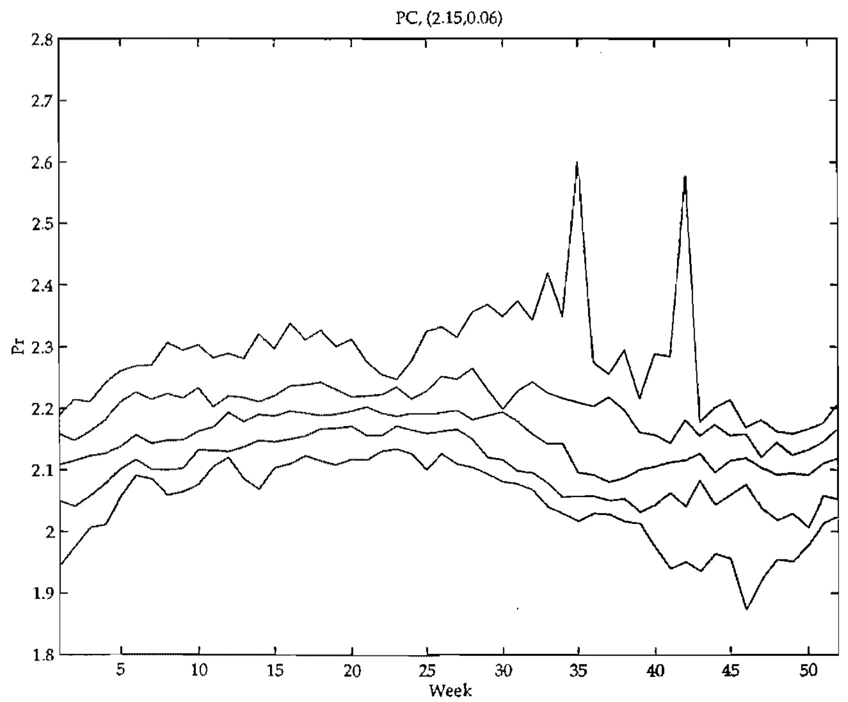

Figure 6.6: PC energy spot price.

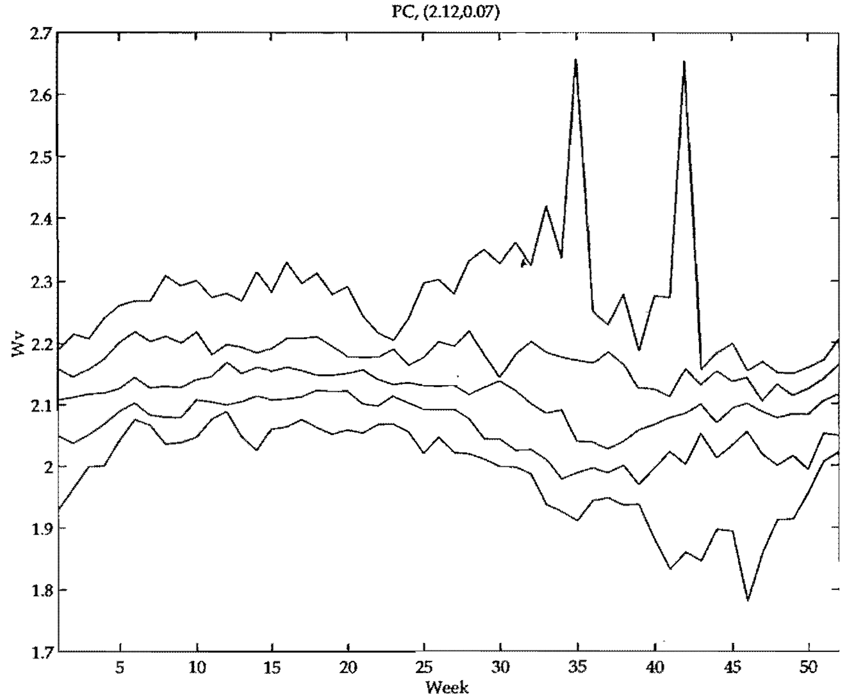

Figure 6.7: PC marginal water value.

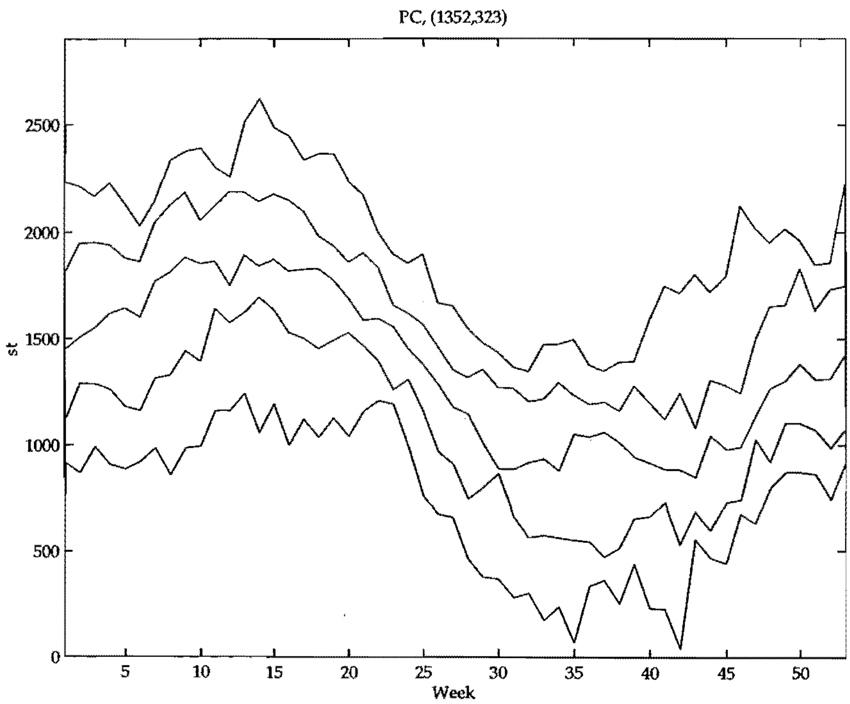

Figure 6.8: PC storage.

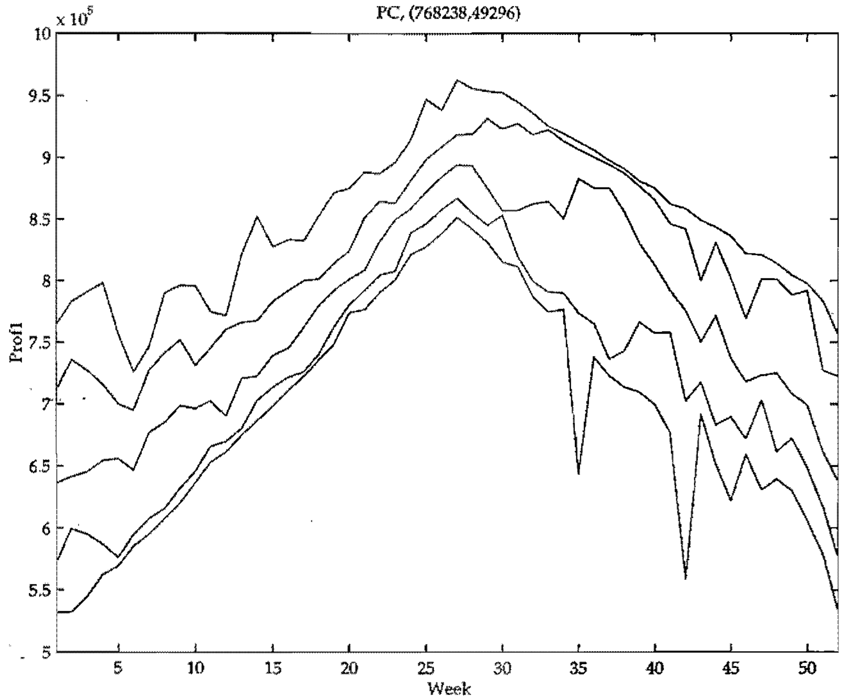

Figure 6.9: PC profit, Firm One.

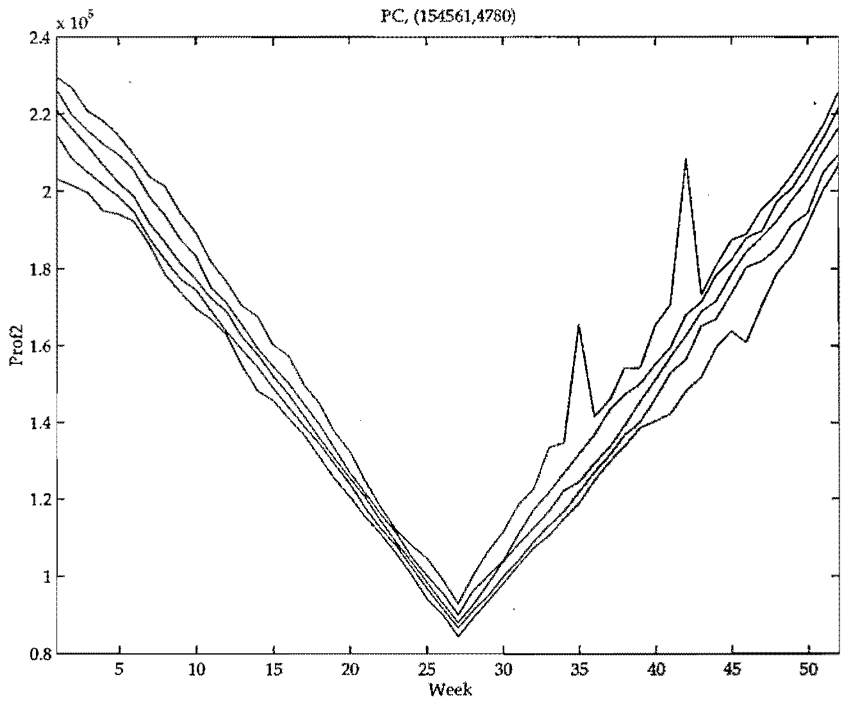

Figure 6.10: PC profit, Firm Two.

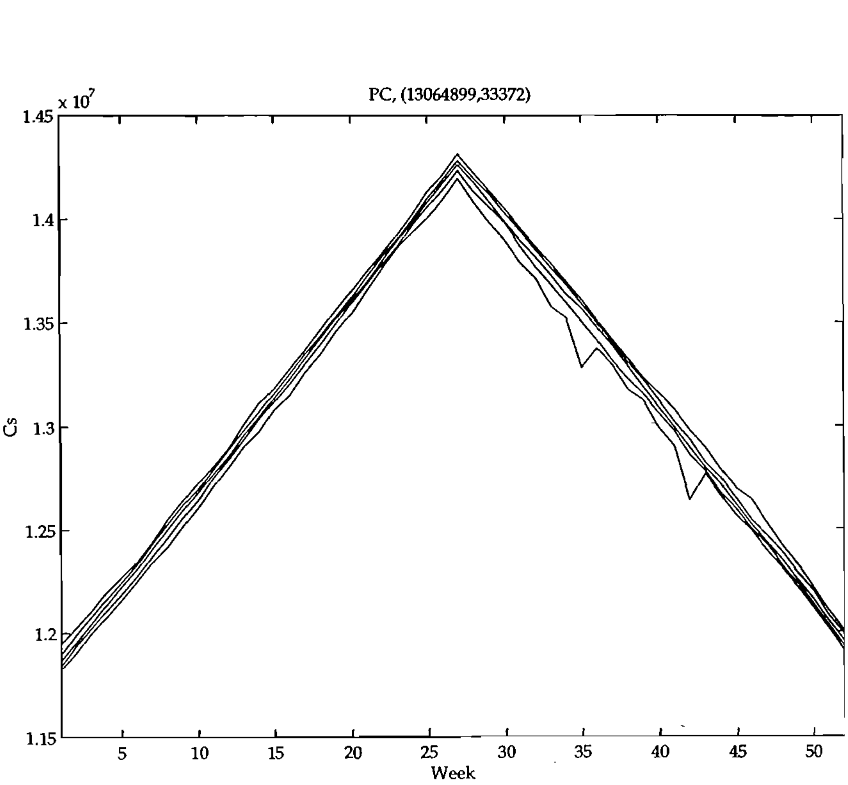

Figure 6.11: PC Consumer Surplus.

6.3 Calibration of the model

According to our theory presented earlier the market results should reproduce the PC results when we have 100% contracting and 100% back-up. Indeed this will be the case when the contracts do exactly match the PC generation. Our model assumes that the contracts are set in advance, and so it is difficult to achieve an exact match. In particular the balance of contracts between the firms and the overall level of output both depend upon the marginal water value. When, such as in a wet year, water is in abundance, and relatively cheap, output will be higher, and the hydro firm (Firm One) will have a greater share of the market. Ideally, the contracts should reflect this, and our back-up do contracts go some way towards re-balancing the contract market.

To achieve a better calibration between the 100% contracts case and the PC case we run the model once in PC mode, and observe the mean marginal water value throughout the 20 year time horizon. We then use this estimate of the marginal water value when setting the contracts for the market runs. The effect of this extra calibration step can be quite marked, as illustrated in Figures 6.12–6.14. The

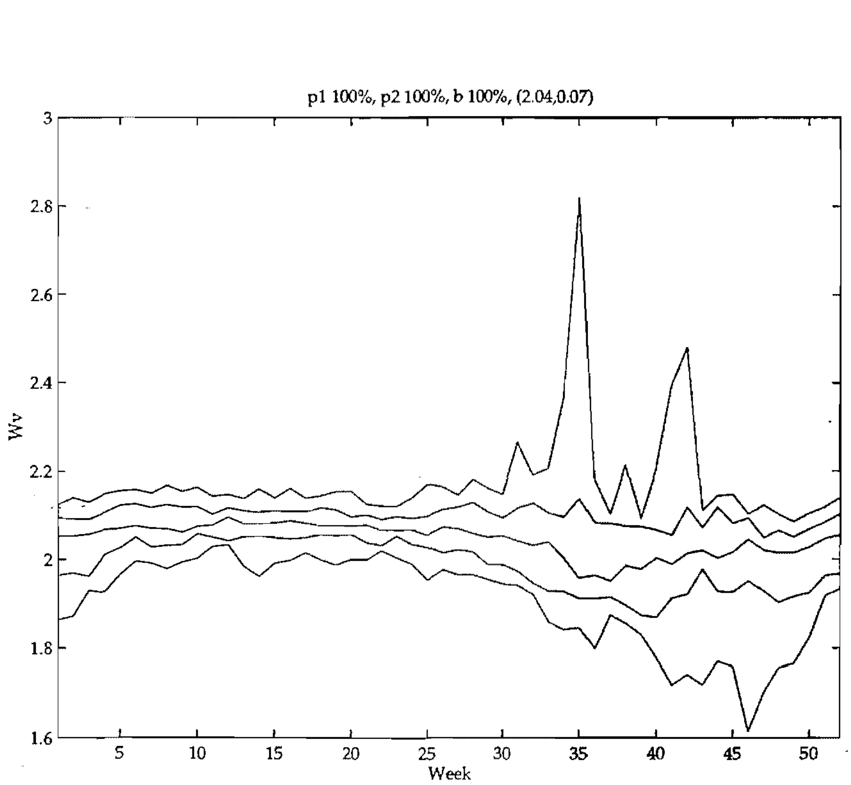

Figure 6.12: Marginal water values, 100% contracts with 100% back-up. Contracts were set assuming the marginal water value would be 3.0. Mean is 2.04.

mean marginal water value for the PC case is 2.12, and when the contracts are set assuming that value the match is quite close. If the marginal water value is erroneously assumed to be 3.0 then the results are quite different. The effect is not restricted to the marginal water values and, as another example, Figures 6.15–6.17 show the corresponding storage trajectories. Again the match is good only when the correct marginal water value is used.

The calibration discussed thus far relates to getting internally consistent results from the model, that is finding the fully contracted level of contracts. Another type of calibration which we need to consider is getting our model to mimic the real system of electricity supply in New Zealand. To this end we have tried to use broadly realistic data in terms of actual demand, station capacities, expected inflow and storage capacity. However our single reservoir model is given significantly more freedom with regards to storage than is available to the New Zealand system.

Major long term storage is available in the Waitaki system in the South Island (approximately 2550 GWh, or 32% of the average annual inflow) and the Waikato

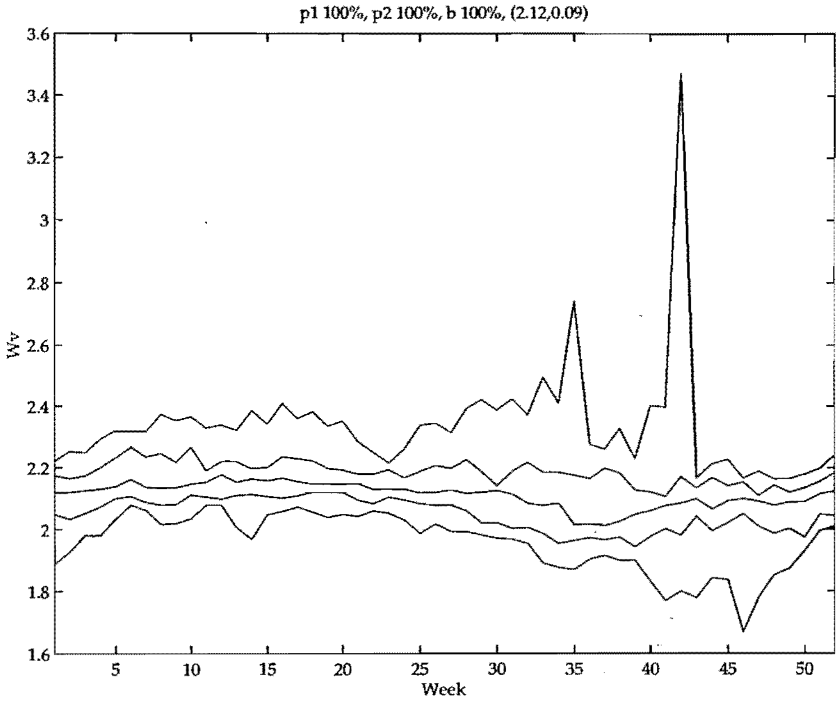

Figure 6.13: Marginal water values, 100% contracts with 100% back-up. Contracts were set assuming the marginal water value would be 2.12. Mean is 2.12.

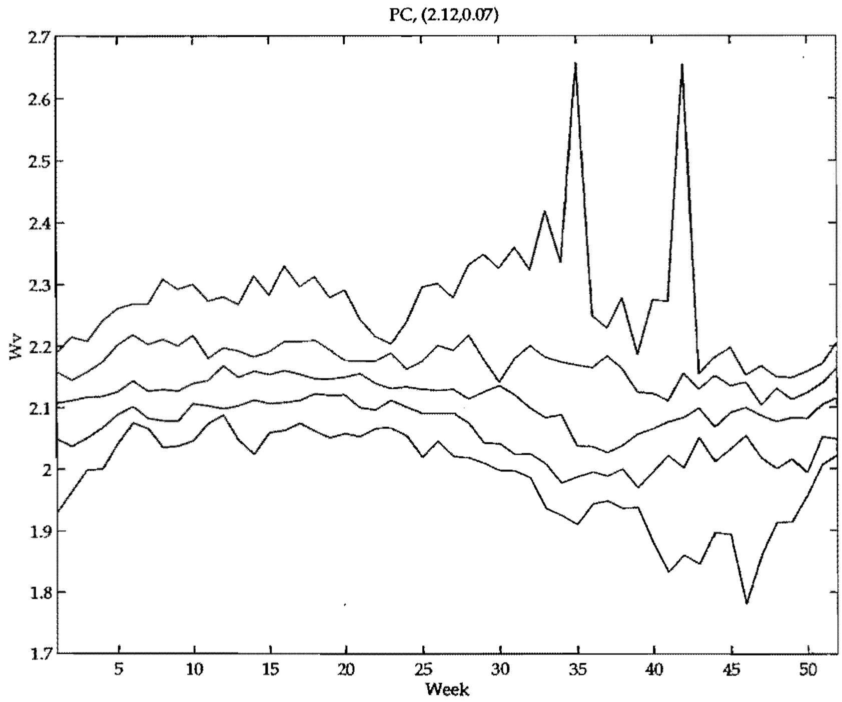

Figure 6.14: Marginal water values, PC. Mean is 2.12

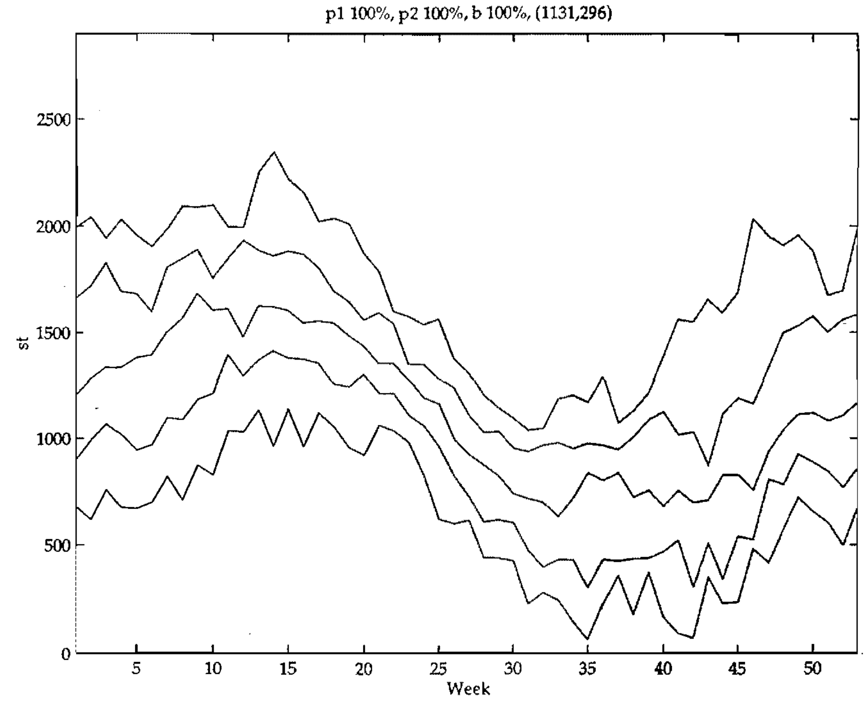

Figure 6.15: Storage trajectory distribution, 100% contracts with 100% back-up. Contracts were set assuming the marginal water value would be 3.0. Mean is 1131.

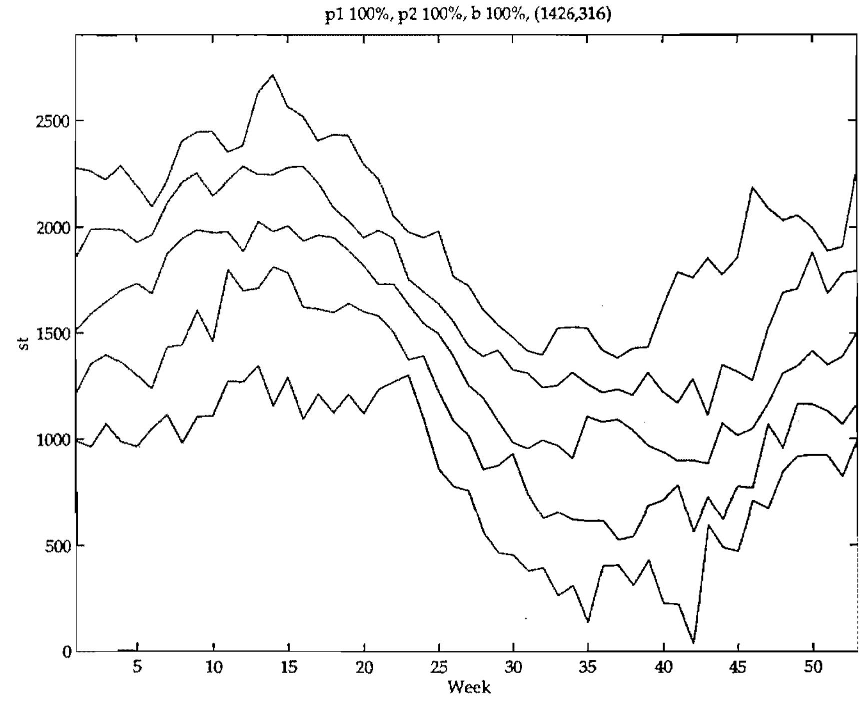

Figure 6.16: Storage trajectory distribution, 100% contracts with 100% back-up. Contracts were set assuming the marginal water value would be 2.12. Mean is 1426.

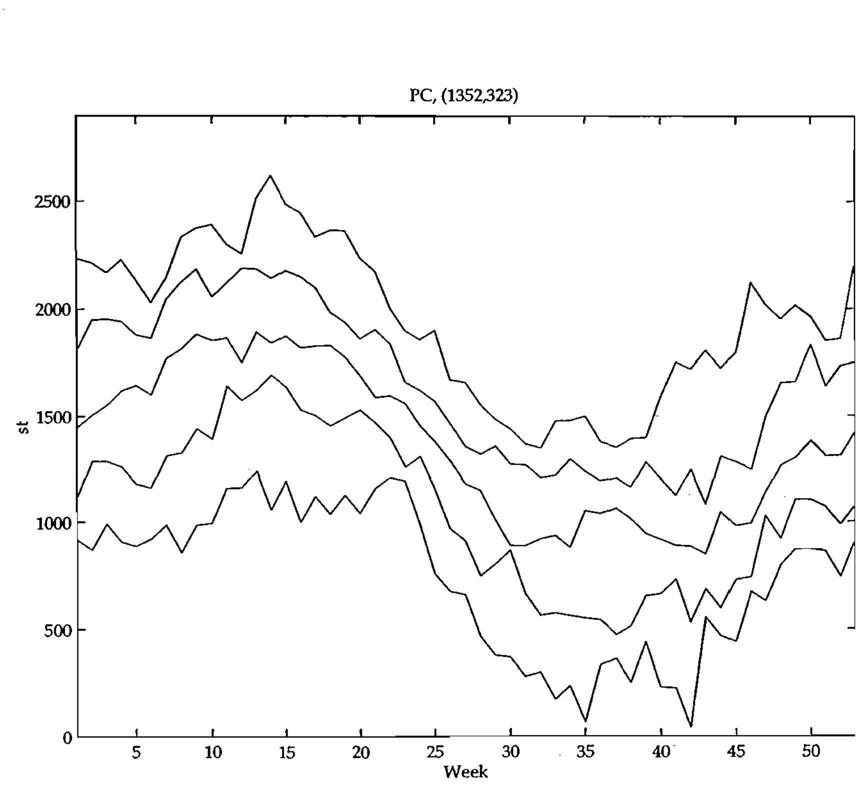

Figure 6.17: Storage trajectory distribution, PC. Mean is 1352

system in the North Island (approximately 650 GWh, or 12% of the average annual inflow), as well as in several smaller reservoirs in both the North and South Islands. The transmission lines linking these systems are of finite capacity, and may often restrict the ability of the system to transfer energy from one place to another. This is particularly relevant to our model when considering the DC link between the North and South Islands. If, as is often the case in winter, the DC link is fully loaded sending power from the South Island generators to the North Island load centres, then it may not be possible to fully utilise all the release capacity in the South Island. Our single reservoir model (with no transmission constraints) does not know this, and may well suggest full release when it isn't possible.

This effect can be thought of in terms of the model thinking it has more freedom near the storage bounds than it really does have. One way to counter this is to simply restrict the size of the reservoir the model has. The expected outcome from this action is to push the storage trajectories to the bounds more often than with a larger reservoir.

To this end we have reduced the reservoir size from 3200 GWh representing the Waitaki and Waikato systems, down to 2900 GWh, a reduction of about 9.4%. The

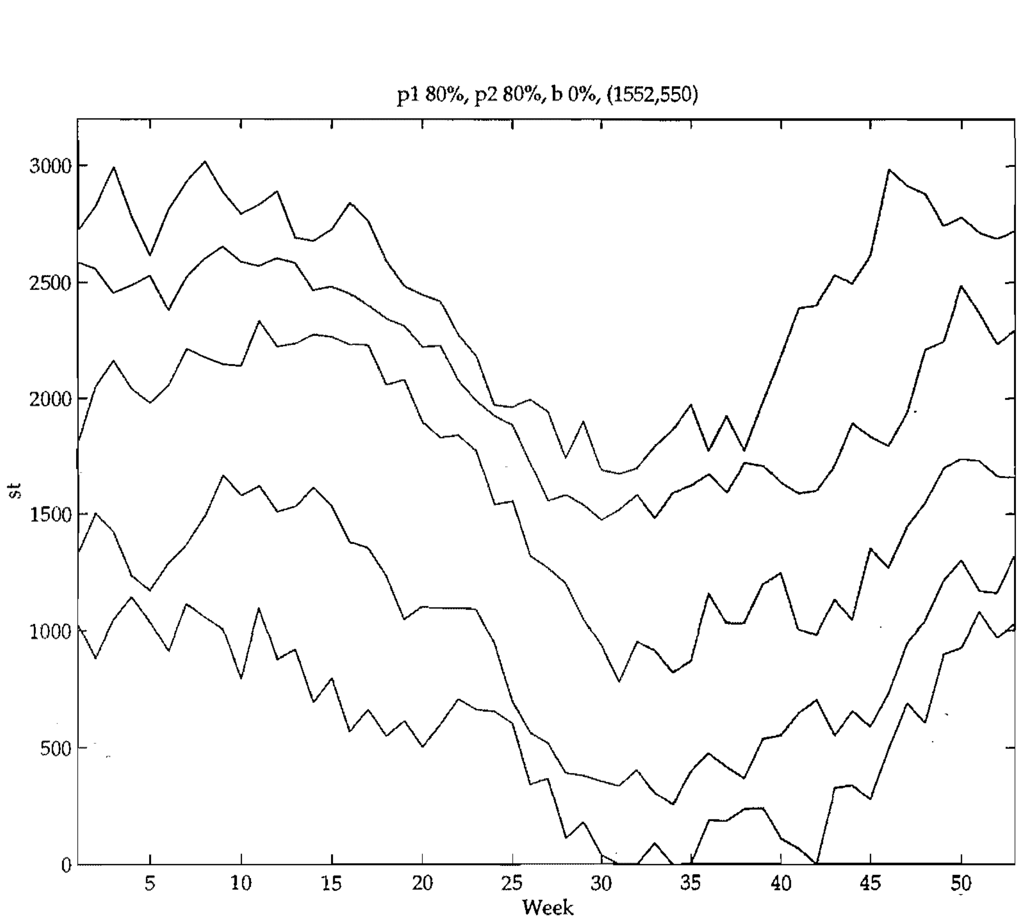

Figure 6.18: Storage trajectory distribution, 80% contracts with no back-up. Storage capacity is 3200 GWh. Mean is 1552.

resulting effect on the storage trajectories can be seen by comparing Figure 6.18 to Figure 6.19. As well as the change in storage trajectories, the reduction in reservoir size affects the fuel cost, the marginal water value and the spot price. The reduction in reservoir size is a reduction in freedom, and comes at a price.

A third factor to be considered when calibrating the model is the interval size on the grid of water values. This is the set of values that we sample at in the single period model. Obviously, finer resolution leads to greater accuracy, but at some computational cost. A question to be asked is what level of resolution is required to produce some acceptable level of accuracy? We have not made a detailed comparison in this study, but instead opted to err on the side of caution with a relatively fine grid[4].

[4]Initial experiments indicated that the difference between ten grid points and twenty grid points was easily observable, but the difference between fifty and one hundred was not. Since one hundred points was within the scope of our computer system we settled for that number.

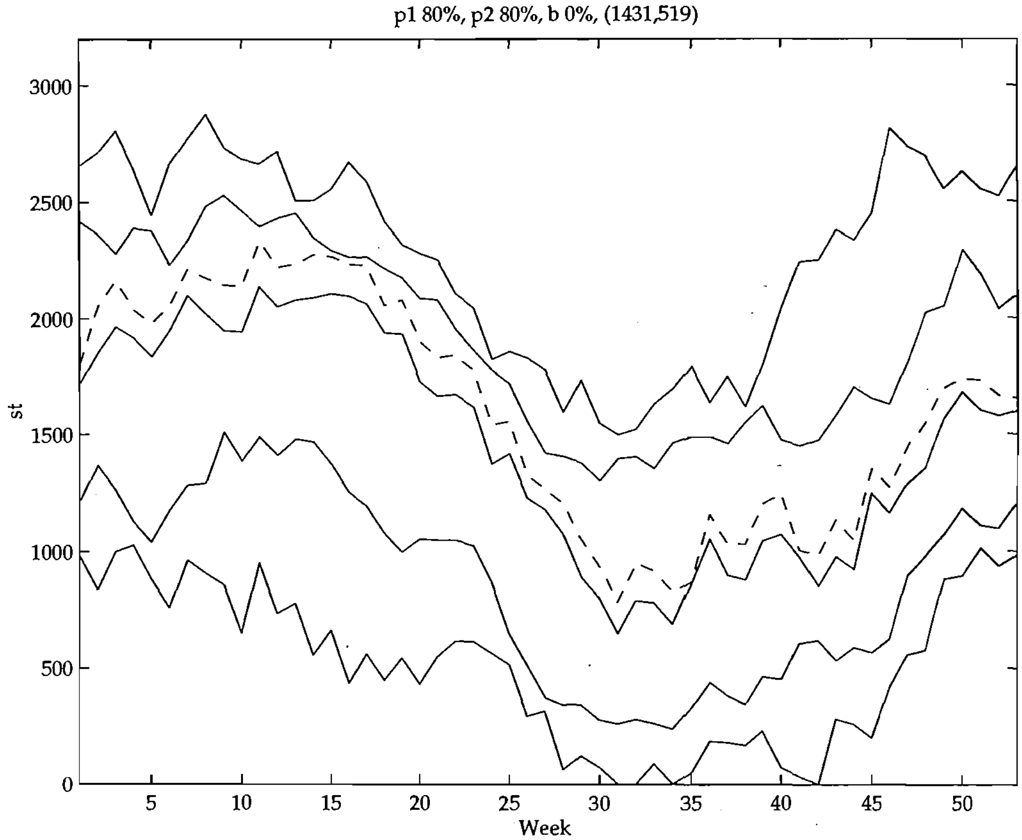

Figure 6.19: Storage trajectory distribution, 80% contracts with no back-up. Storage capacity is 2900 GWh. Mean is 1431. The mean trajectory from Figure 6.18 is shown as a dashed line for comparison.

A fourth consideration is the number of weeks for which we should run the single period model. While there is a great difference between the peak winter load and the lowest summer load, the variation from week to week in a given season is much less extreme. Our long term model takes a set of weeks representing the range of loads throughout the year, and interpolates to get values for the missing weeks. This may mean that we run the single period model for only two weeks, mid-winter and mid-summer, and interpolate for the rest, or we may sample every four weeks, or even every week. The sampling process is expensive computationally, and without accurate load data it seems pointless to go to the extreme of sampling every week. In fact if we are aiming to investigate trends rather than to come up with accurate quantitative results then the opposite extreme of one winter and one summer period is adequate.

6.4 Results for the *ec2* model

Figure 6.20 shows the means and standard deviations of the total generation for Firm One for a range of contract levels. The contracts range from 50% to 120% for each firm. However the results are plotted against the total contract amount rather than the percentages. The reason for this is that our earlier analysis (Chapter 3) demonstrated that the important factor is the total contract amount rather than the percentages. This also presents a more consistent picture when, as is the case here, the firms are of rather uneven sizes.

Each point in Figure 6.20 is a summary of a twenty year simulation run for a particular combination of contracts. The PC amount is plotted as a circle rather than a plus sign. Appendix A contains the complete set of summary graphs for generation, price, water value, storage level, profit and consumer surplus. For each variable we present graphs for three levels of back-up contracting and three levels of elasticity.

Recall that in Chapter 4 we observed in our study of the PDI that as back-up increased it became less important which firm had contracts. For low levels of back-up we observed that Firm One's contracting had greater influence on price distortion than did Firm Two's. For high levels of back-up this was no longer the case. An interesting question, then is to ask if this carries over to the simulation results. If it does, then there may be good grounds for leaving out a lot of the added complications of a hydro simulation model and instead basing further research on single period models and thermal systems. If the results do not carry over from the single period to the simulation, then we have empirical evidence that the hydro system is bringing added complexities to the market, and it will be important to keep that in mind in any future work.

Given that our single period observations were centred around price distortion, it would seem likely that if similarities are to be found between any of the simulation results and the single period results, then they will be found in the measures of the energy spot price. These are shown in Figures A.36–A.42.

As was the case in Chapter 4, one striking feature of these graphs is that there are bands of points. Within each band the contracts for Firm One are constant, and those for Firm Two are increasing. While we see to some extent that as back-up increases the bands tend to merge into one, indicating that the market is less sensitive to the original allocation of contracts, the merging is not as extensive as it was in Figure 4.21. The most likely explanation seems to be that it is a result of the back-up contracts being based on the *expected* marginal water value, not the *actual* marginal water value. We set the back-up contracts assuming the marginal water value would be 2.12, and only when the actual value is the same as this expected value do we get a perfect match.

For the firms in our Cournot model of the energy market generation is the controllable factor. Figures A.1–A.7 show the total generation for Firm One. This is broken down into thermal (plus run of river hydro) in Figures A.8–A.14 and hydro in Figures A.15–A.21. As expected from the single period model and from intuition, generation increases as contracts increase for the firm[5]. With no back-up, and moderate to high levels of contracting we see that Firm One's output decreases as Firm Two's contracts (and hence their output) increases. This is the normal behaviour expected, and stems from the negative slope of the reaction functions, as discussed in Chapter 3.

Note that although the generation is very sensitive to the contracts, it is less

[5]Recall that along each band the contracts for Firm One are constant, and those for Firm Two are changing.

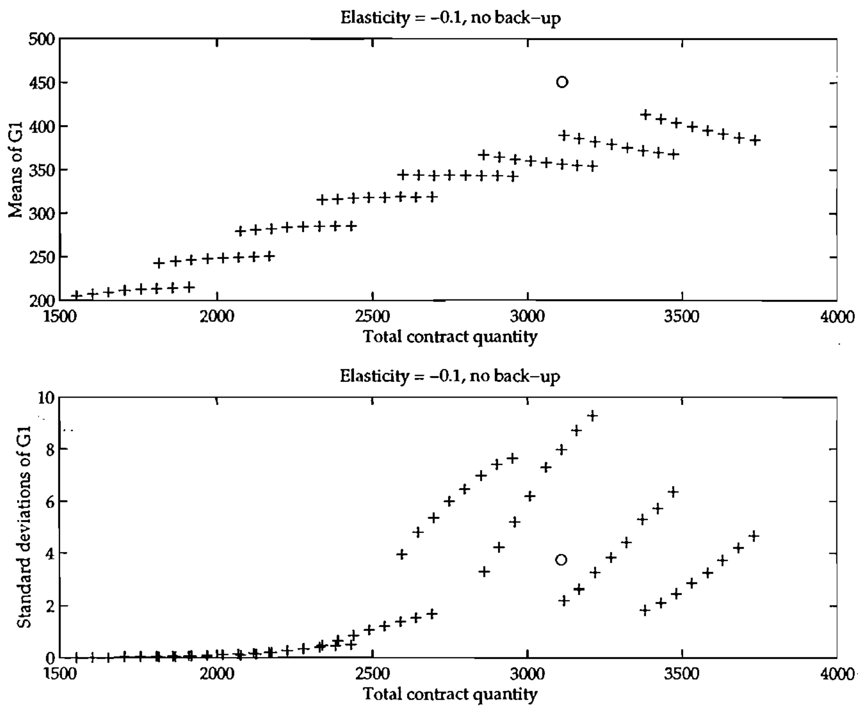

Figure 6.20: Means and standard deviations of total generation, Firm One. Elasticity = -0.1, no back-up.

so to our choice of demand elasticity, within reason. At low contracting levels the sensitivity to elasticity is enhanced. Note also that there is a much greater change between elasticities of -0.1 and -0.33 than there is between -0.33 and -0.8. However, checking the energy spot price in Figure A.36 indicates that such a situation is unlikely to occur in practice, especially on an ongoing basis. Energy spot prices of 50 cents per kiloWatt-hour are likely to bring new entry at the least, and most likely Government intervention as well. This should not be seen as a sign that our model has failed, but more simply that we are operating it outside its intended range. In particular our point estimate of the demand was at less than five cents, and to extrapolate to more than 50 cents is not wise. With 100% contracting and 100% back-up the market output exactly matches the PC levels[6].

[6]100% contracting has the same total contract quantity as the PC point, shown as a circle in the graphs.

If we ignore the extreme parts of Figure A.1 then we can observe that the standard deviations of Firm One's generation is decreasing as contracts increase. Conversely the standard deviations of the thermal and hydro components is increasing as contracts increase. At low levels of contracting almost all the variation is in hydro output, due to inflow variations. As contracting increases it becomes more important for the firm to meet the contract target, and the standard deviations of total generation fall. However, the hydro inflow is still subject to random variations, and the firm uses its thermal capacity to offset these variations, leading to an increase in variation in both hydro and thermal generation. The corresponding storage plots of Figures A.64–A.70 reinforce this idea, as storage variation is also decreasing as contracts increase, showing once again a tendency to let release follow inflow.

Another interesting point regarding the graphs of standard deviation of total generation is that it peaks at around 2000 MW, and drops off on either side. It seems that the reason for increased variation in output is the shape of our marginal cost curve. Our model has Manapouri as a run of river hydro station with capacity of 570 MW. This corresponds to just under 100 GWh per week, which is the level of the large flat section, especially seen in Figure A.11. Over this region the Manapouri is at full capacity, and Huntly is not yet running. This restricts the abilitiy to use the thermal capacity to manage inflow variations, and hence total variation increases.

Firm Two's generation is shown in Figures A.22–A.27. As expected it increases as Firm Two's contract amount increases (along each band) and decreases as Firm One's contracts increase (between bands). For the most part it is above the PC level, a result of our initial contract allocation which has left Firm Two over contracted. Again it exactly matches PC for 100% contracting and 100% back-up. For the cases where there is no back-up the standard deviations are decreasing as contracts increase, showing the firm's desire to more closely match output with contracts. With back-up, the situation is less clear, especially for low contracting levels.

Since our demand curve has a negative slope, increased generation leads to decreased prices. Since generation increases with contracts, price must decrease. At 100% contracting and 100% back-up the spot price exactly matches that of PC.

However the standard deviations of price follow the same trend as total market generation, not the opposite ones. Whereas an increase in generation implies an increase in price, an increase in variance of generation implies an increase in variance of spot price.

Spot price is affected by the elasticity of demand and, as with generation, the effect is most pronounced at low levels of contracts. For moderate to high levels of contracting the difference in spot price for the elasticities considered is less than their standard deviations.

The profit[7] (Figures A.43–A.56) of each of the firms decreases with increasing contracts, and in particular it decreases with increasing contracts of the other firm. It is clear from the graphs that for a firm to make profits above the PC level requires, in most cases, that both firms be under contracted. Note that for the purpose of these profit calculations the contracts were valued at the spot price. This situation could conceivably arise in a stable market where the spot price from one year fed through to the contract strike price for the following year.

An interesting use for the calculations in these graphs would be to value the contracts. The increase in profit between one contract position and another should be a good estimate of the value to the firm of moving from the one position to the other. That this value is negative for increasing contracts indicates that the firms should be selling contracts at a premium. Similar analysis of the consumer surplus (Figures A.71–A.77) indicates that the consumers should be willing to pay a premium to get the contracts[8].

Also of note is the fact that the standard deviations of Firm One's profit is lowest at low contract levels, precisely when their profit is the highest. From a portfolio analysis perspective this is ideal, having high profit and low variance. However the same is not true for Firm Two, and neither is it so for the consumer surplus. Together these imply that the equilibrium point is likely to be above the 50% level which we used as our lower extreme. Again see Batstone (1997).

As discussed above, at high levels of contracting the firms will be trying to generate close to their contracted amounts. For Firm One, with the hydro reservoir, this implies that any shortfall in inflow will be made up for with thermal

[7]We only consider the variable part of the profit, ignoring fixed costs and payments that are beyond our immediate control.

[8]See Batstone (1997) for an equilibrium analysis based on this idea, and also Powell (1993).

generation, at their marginal thermal cost. Thus the marginal water value should approach this marginal thermal cost as contracts approach 100%. At low levels of contracting the firm will be more indifferent to spill[9], and the marginal water value will be lower. This is precisely the behaviour shown in Figures A.57–A.63.

The standard deviations of the marginal water values will tend to be low at both extremely low and extremely high levels of contracting. At low levels water in storage is of little value to us as we are restricting supply considerably anyway. At high levels of contracting the marginal water value is at the marginal thermal cost, regardless of inflows. In between the marginal water value will vary depending on the inflow levels.

Storage is driven by marginal water values. For low contracting we have a low marginal water value and a high mean storage level[10]. As contracting increases the mean storage level decreases to around the PC level, but then seems to rise again. The reason for this is not clear, and it would be of interest to conduct an experiment where contracting was allowed to range much higher than the levels we have considered to see if the effect continues.

In summary, with increasing contracts we can expect the following:

- increasing output
- decreasing energy spot price
- decreasing profit
- increasing consumer surplus
- increasing marginal value of water
- decreasing storage.

[9]At low levels of contracting the firms will both be withholding generation from the market in order to extract monopoly profits. An extra unit of generation under these circumstances would actually decrease profits, so the only economic use for the extra water is to offset thermal costs. However with less total generation there is less thermal generation to offset, and the chances of finding a use for the water before the reservoir is full are less. All of this implies that the marginal value of water is less at low contract levels than it is at high contract levels.

[10]This has an associated increase in risk of spill, leading to a decrease in the hydro generation. This is clearly shown in Figure A.15 and Figure A.64.

6.5 Results for the *ww* model

The same general trends we observed for the *ec2* model are evident in the *ww* model. However it is interesting to compare the levels in the *ww* model with the corresponding results from the *ec2* model. The *ww* results are shown in Figures A.78–A.88.

Recall that the allocation of stations to the two firms, and to the fringe, is shown in Table 4.2. In this model Firm One has only the major hydro with storage. It does not have any thermal, with Huntly being in Firm Two, and no run of river hydro, that all being in the fringe. As a result their thermal generation is zero, and the hydro generation matches their total generation. Also, the total capacity shared by the two firms is considerably less than under the *ec2* option, and hence the PC generation from the firms is lower, and the range of contract quantities (corresponding to 50%—120%) is less. (The fringe generation is up, since they now have both Clutha and Manapouri.) Also note that Firm Two actually holds more low merit order capacity than Firm one, as Huntly is priced below the assumed marginal water value of 2.12. In terms of the summary graphs, the large (horizontal) gaps between the plotted points are for increasing contracts of Firm Two, and the small gaps for increasing contracts of Firm One. This is the opposite of what is presented in the results for the *ec2* model. Thus in Figure A.81 G2 is increasing with increasing contracts for Firm Two, and decreasing with increasing contracts for Firm One, as expected.

The hydro generation of Firm One remains fairly constant, although with slightly lower output and lower standard deviation than in *ec2*. Firm Two, however, is consistently generating at above their PC level, and as a result the energy spot price is consistently less than PC. The cause for this somewhat counter-intuitive behaviour is the lower than expected marginal water value. The contracts were set assuming a marginal water value of 2.12, but as Figure A.86 shows, the actual value in the simulation was most times less than 1.0. Under PC, low water values would imply a shift of generation from thermal to hydro. The market model, however, has contracts for Firm Two at well above this PC level, even when it is below the PC level for a water value of 2.12. Hence Firm Two chooses to generate at a level close to their contract amount, and well above the PC level.

The lower hydro output appears to be a result of the low marginal water values and the correspondingly high storage levels, leading to increased risk of spill.

6.6 Other interesting results

An interesting effect can be seen if we allow the contracts to be set differently in winter than they are in summer. In this section we consider two cases. In the first case we set summer contracts at 100% and winter contracts at 75%. In the second case we set summer contracts at 75% and winter contracts at 100%. In both cases the back-up is 100%.

The results from the first case are shown in Figures 6.21–6.24. What we notice is that the expected seasonal variations are reinforced by the combination of contracts. The spot price is low in summer, aided by full contracting, and it is high in winter, pushed even more so by the low contracting. In addition the water values are slightly higher, and the storage trajectory is lower and less extreme, with decreased chance of spill.

Conversely, for Figures 6.25–6.28 the contracts oppose the natural seasonal variations, and do so to such an extent that we have a higher energy spot price in summer than we do in winter. The water values are lower, and the storage trajectory is more extreme, leading to increased chance of spill.

Such a situation may seem extreme, but it does seem reasonable to expect higher percentage contracting in winter than in summer.

6.7 Conclusions

We considered contracts ranging from 50% of the PC levels to 120% of the PC levels. Through careful calibration we achieved an almost perfect match between PC and 100% contracting, which is exactly what we had hoped for. This calibration involved adjusting the parameter describing the expected marginal water value to equate it with the mean marginal water value seen under PC. This affected the initial contract allocation. In addition, we restricted the size of the storage reservoir to help account for the bias from only modelling one reservoir.

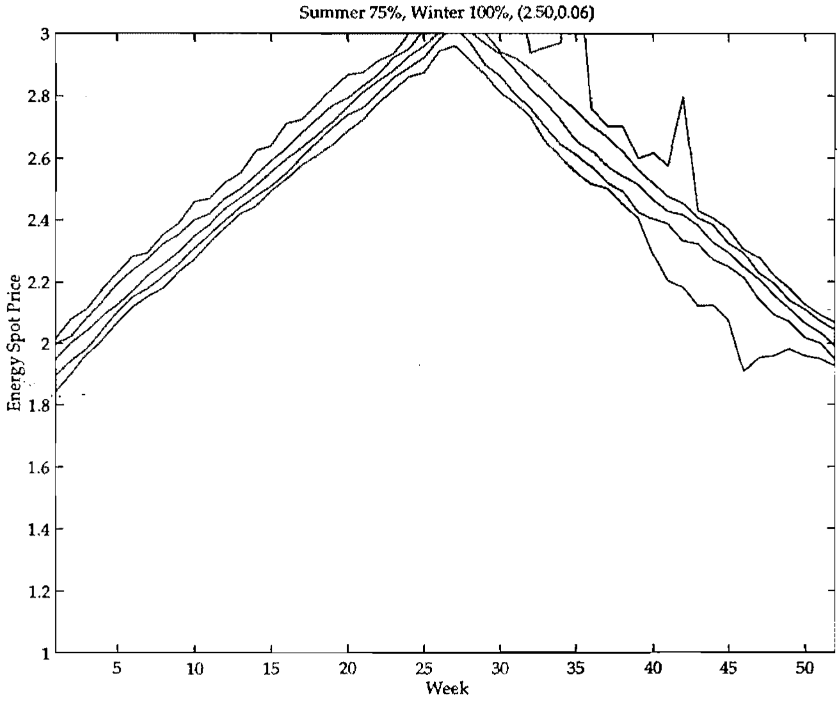

Figure 6.21: Energy spot price. Summer 100%, Winter 75%, 100% back-up.

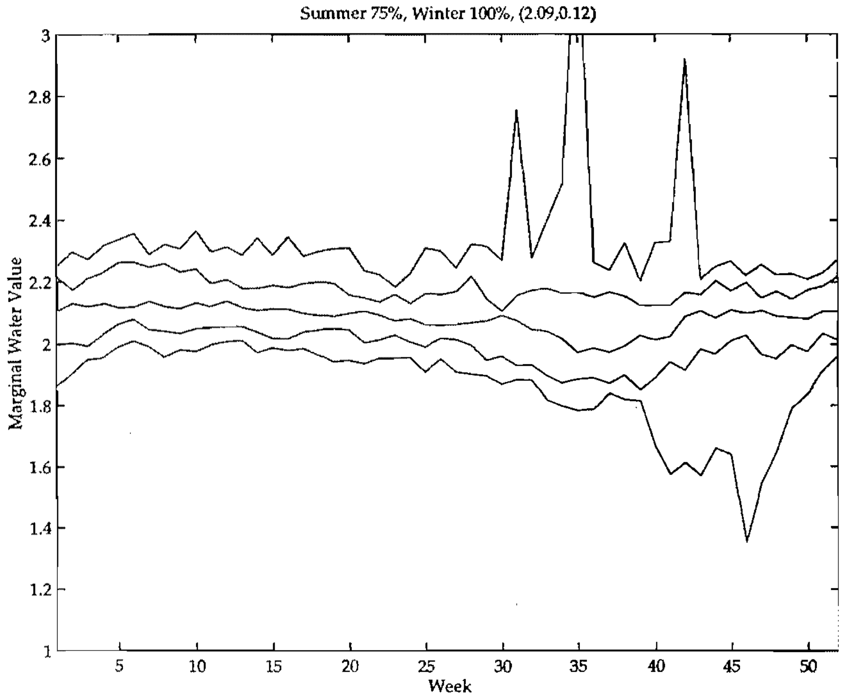

Figure 6.22: Marginal water value. Summer 100%, Winter 75%, 100% back-up.

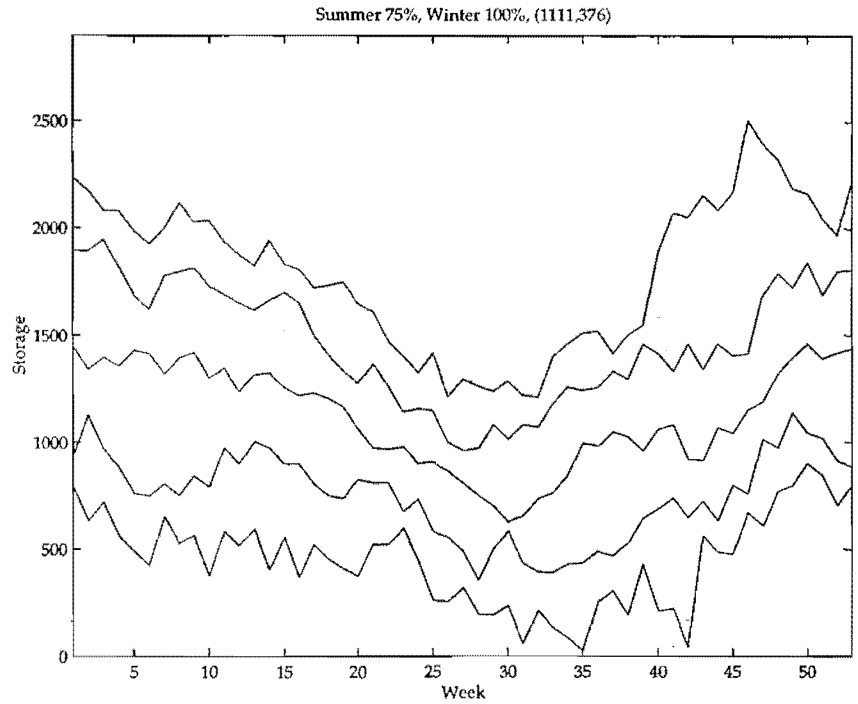

Figure 6.23: Storage. Summer 100%, Winter 75%, 100% back-up.

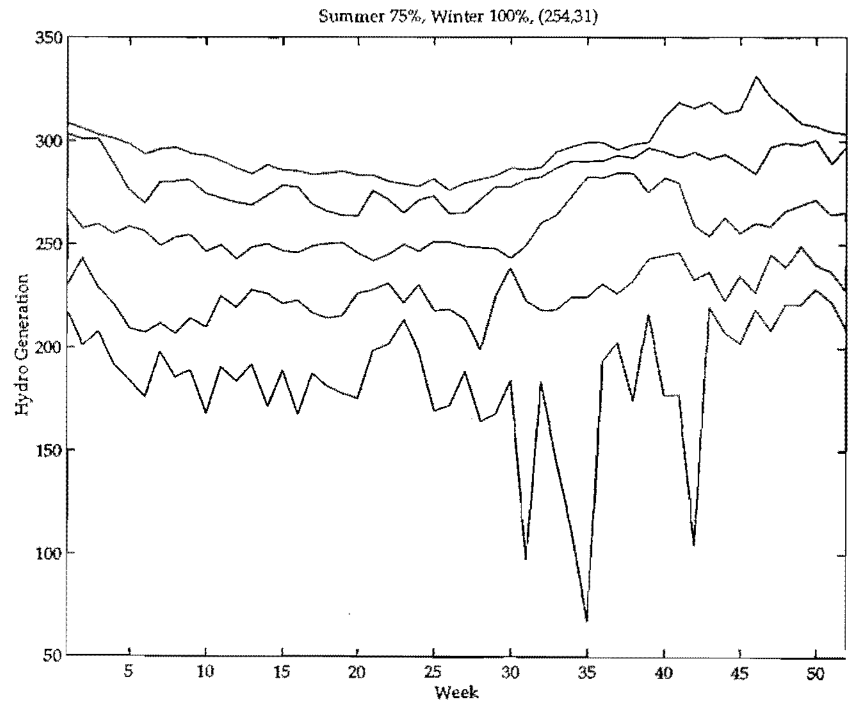

Figure 6.24: Hydro generation. Summer 100%, Winter 75%, 100% back-up.

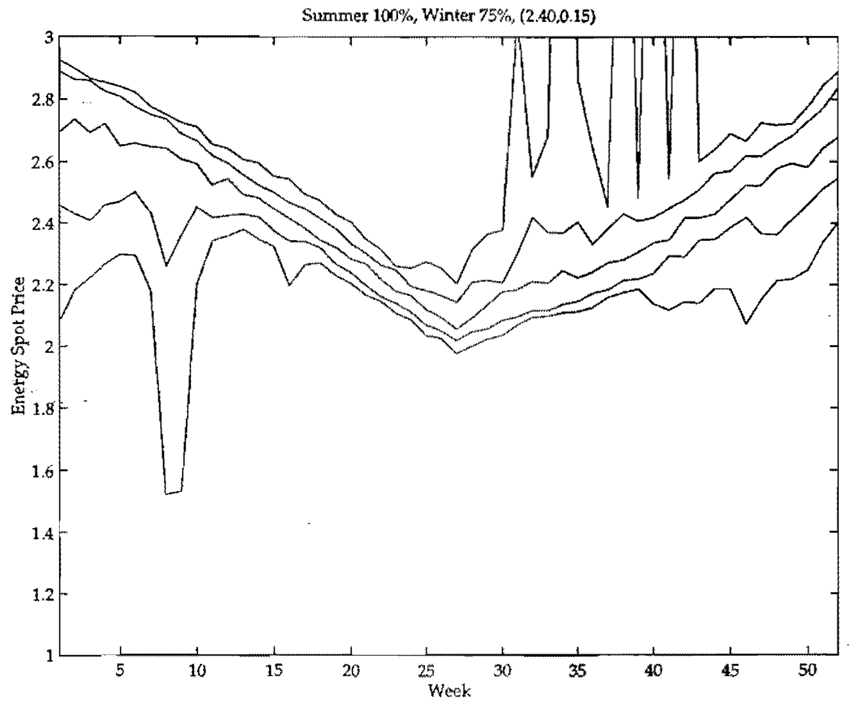

Figure 6.25: Energy spot price. Summer 75%, Winter 100%, 100% back-up.

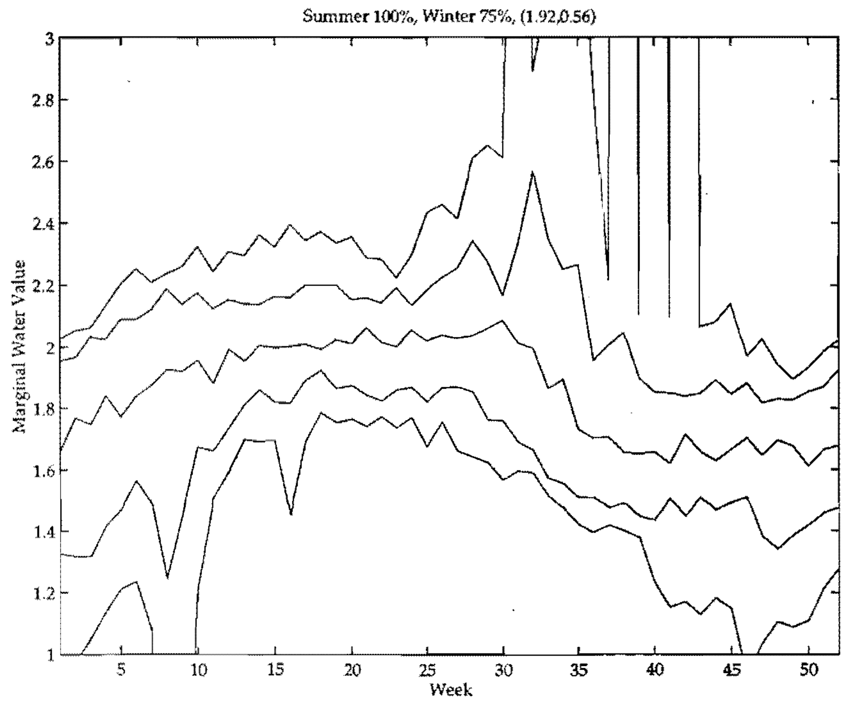

Figure 6.26: Marginal water value. Summer 75%, Winter 100%, 100% back-up.

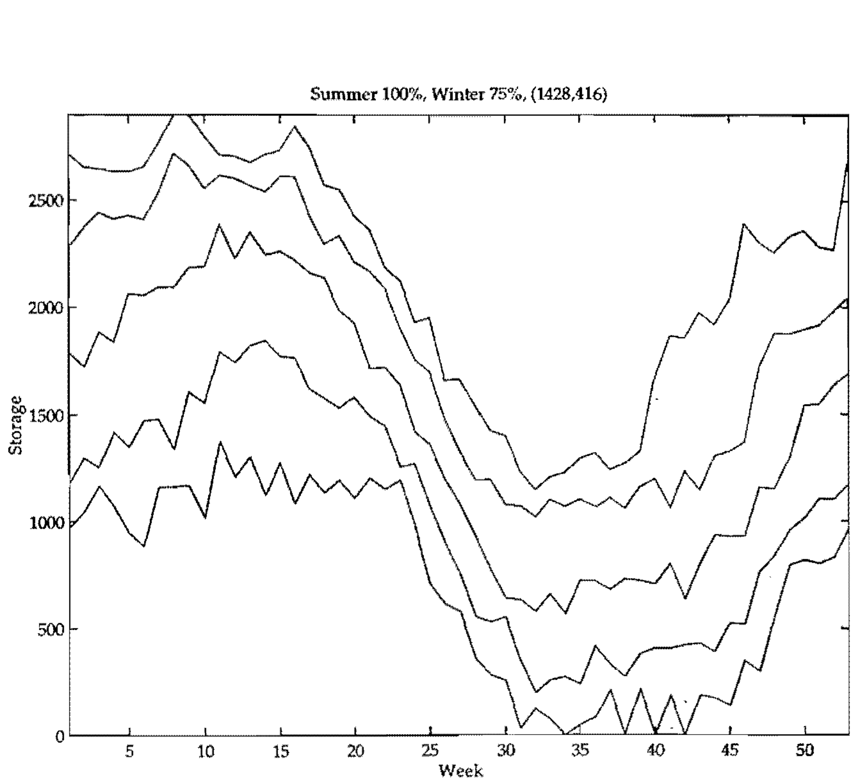

Figure 6.27: Storage. Summer 75%, Winter 100%, 100% back-up.

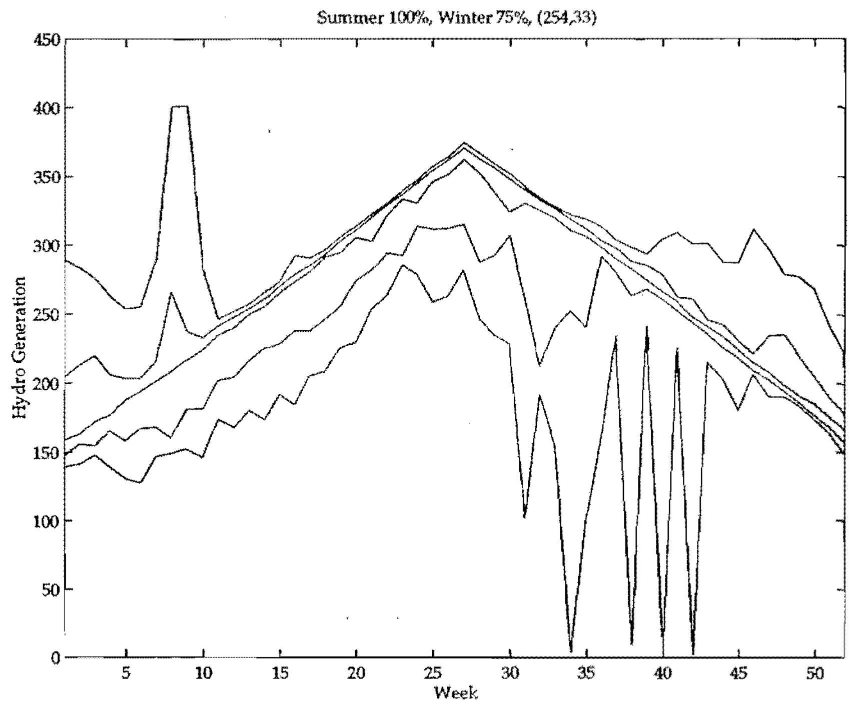

Figure 6.28: Hydro generation. Summer 75%, Winter 100%, 100% back-up.

We have presented a range of results from the simulation runs. concentrating on the *ec2* model, mostly because it best represents the split of companies in the NZEM. The range of elasticities we studied is from -0.1 to -0.8, which covers most reasonable estimates of the medium term elasticity of demand. We summarised our conclusions by saying that with increasing contracts we can expect the following:

- increasing output
- decreasing energy spot price
- decreasing profit
- increasing consumer surplus
- increasing marginal value of water
- decreasing storage.

Comparison with the *ww* model in which Firm One has only hydro stations, not thermal, indicated that the *ww* model lead to lower energy spot price and higher output than under *ec2*. higher storage trajectories lead to increased risk of spill.

Finally we presented a paradoxical situation where low contracts in summer and high contracts in winter lead to the summer energy spot price being higher than the winter one.

Chapter 7

Conclusions

This chapter summarises the main results and ideas presented in this thesis, and outlines possible areas for future study.

This thesis combines two important areas of energy sector modelling. They are the modelling of competitive electricity markets, and the optimisation and simulation of hydro reservoir systems.

7.1 Single Period Model

We have modelled the NZEM as a simple Cournot market trading only in energy. This allowed us to keep the single period model relatively simple, but still captured much of the dynamic behaviour of the energy spot market. The basic Cournot model is extended to allow consideration of contracts, and much of this thesis is studying the effects of those contracts.

Although we settled on the reasonably simple Cournot oligopoly model as a basis for our single period model, other models may be better suited to the NZEM as it has now evolved. In particular the bidding process of the NZEM might be better modelled as a multi-stage game.

To be more thorough and more realistic the spot market model should also consider the reserve market, and should account for transmission constraints and optimal nodal pricing.

As long as the spot market model produces downward sloping demand curves for release, we will be able to incorporate it into the DDP framework as presented in this thesis, and we will have an effective system for optimising reservoir management in the NZEM.

In Chapter 3 we developed the theory behind our Cournot model, and proved the existence of unique equilibria for both linear and constant elasticity demand curves. The proofs also enabled us to show that our Cournot model will produce the downward sloping demand curves for release that are required for the DDP optimisation.

Our experiments with the single period model lead us to conclude that the single biggest effect on price distortion is the level of contracting, with the generation rising and energy spot price falling as contracts increase. However the elasticity of demand also, predictably, has a large influence on the level of distortion.

With full back-up, the level of price distortion decreases monotonically with increasing market contract quantities. With less than full back-up the price distortion still decreases, but the contracts of the larger firm have a much greater impact than do those of the smaller firm.

The market power of a firm is influenced not only by their total capacity, but also by their portfolio, that is by their capacity at different price levels. Firms with low merit order plant are better able to influence the market than those with higher marginal cost plant.

We found that the choice between linear and constant elasticity demand curves for the single period model is somewhat arbitrary. Both give believable results, but the linear demand curve leads to analytic solutions, where the constant elasticity curve requires a numerical solution method. However the numerical solution converges very quickly, and both approaches are tractable.

The *ww* break-up option has lesser price distortions than the *ec2* model. The reason seems to be that in the latter model one firm is roughly twice the size of the other, and has much greater ability to influence the market. In the *ww* model are more closely matched in size.

We also briefly studied the situation where one of the firms acts as a price taker, and noted that this had the very desirable effect of moderating distortions. However it did sometimes lead to non-monotonicities, and hence difficulties with the medium term model.

7.2 Dual Dynamic Programming and Reservoir Management

We have presented the DDP approach to reservoir mangement and explained how it closely relates to standard economic processes and principles. We have shown that DDP is equivalent to the process of adding demand curves for different time periods, and that as long as the individual demand curves are downward sloping the process will yield an optimal water value surface. That water value surface is a concise representation of the marginal value of water at any given storage level and for any point throughout the time horizon.

While we have presented a single reservoir model here, DDP can be extended to two or more reservoirs, although this comes at some considerable computational expense (the curse of dimensionality). For the NZEM, and in conjunction with incorporating transmission constraints into the single period model, a two reservoir model may offer some gain in accuracy.

7.3 Simulation Results

The simulation runs have clearly demonstrated the strong link between contracts and generation levels. This was expected from the theory we developed in Chapter 3. The simulations have also demonstrated the sensitivity of the model to changes in elasticity of demand. If a model such as this is to be implemented then it would be wise to further refine the demand side description, if only by trying to better estimate the true demand elasticity.

We summarised our conclusions from the simulation runs by saying that with increasing contracts we can expect the following:

- increasing output
- decreasing energy spot price
- decreasing profit
- increasing consumer surplus

- increasing marginal value of water
- decreasing storage.

The tests between the two market structures we considered indicate that the *ww* structure, with Firm One owning only the Waitaki and Waikato systems, with Firm Two having all the thermal and geothermal plant, and the fringe having Clutha and Manapouri, gave higher output and lower prices for every level of contracting.

The case where the market is more heavily contracted in winter than in summer lead to the interesting situation of higher spot prices in summer than in winter.

When the market is sufficiently under-contracted there is an increased likelihood of spill, even though energy is being withheld from the market. This can mean that a generator finds it optimal to spill even though they are not running at full capacity, a very unattractive proposition.

However with reasonable levels of contracting it appears that the wholesale electricity market is not too far away from perfect competition.

7.4 Future work

A major dynamic factor we have not captured in our model is the way in which contracts are negotiated over time. It is reasonable to expect that prices today will influence both the price consumers will pay for contracts (in the medium term) and the overall demand for electricity (in the longer term). The renegotiation of contracts is considered by Batstone (1997), and in his forthcoming PhD thesis.

Adjusting our model to allow feedback from the spot prices to the future demand and contracts would certainly add an extra dimension of reality to our work.

Our DDP framework would work equally well with other single period models, and now that the New Zealand situation is better known it would be worthwhile investigating alternatives to our Cournot model. A model which better represents the dynamics of the demand side may add some further insights. Also worth considering would be the modelling of the transmission system in some form, and the parallel market for reserve energy.

If the transmission system were to be modelled, it would be worthwhile moving to a two reservoir DDP model at the same time. Such a model, if well planned, should be well within the capabilities of todays computer systems.

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Appendix A

Multiple Period Results – Summary graphs

A.1 Results for the *ec2* model

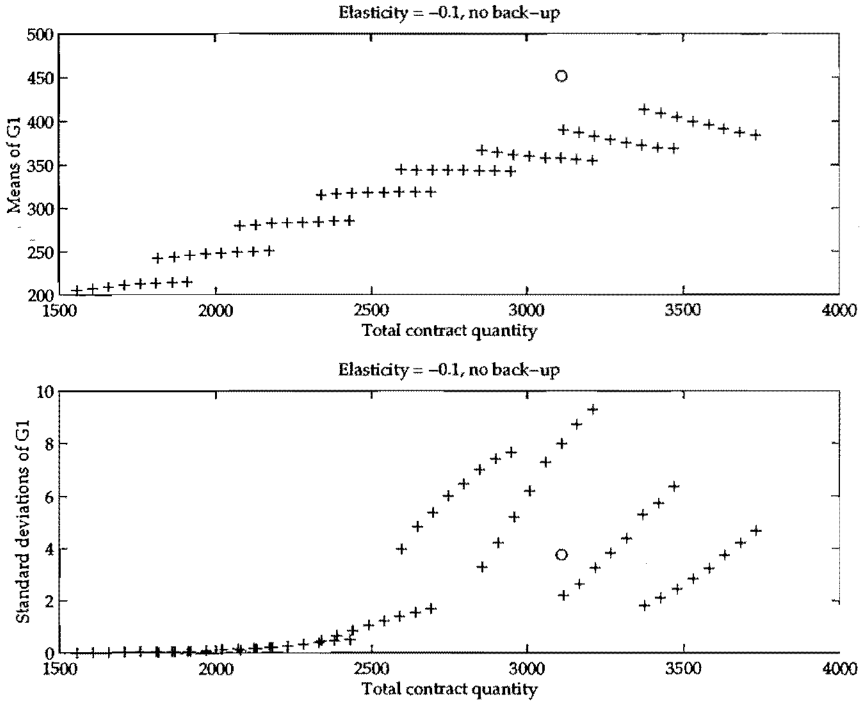


Figure A.1: Means and standard deviations of total generation, Firm One. Elasticity = -0.1, no back-up.

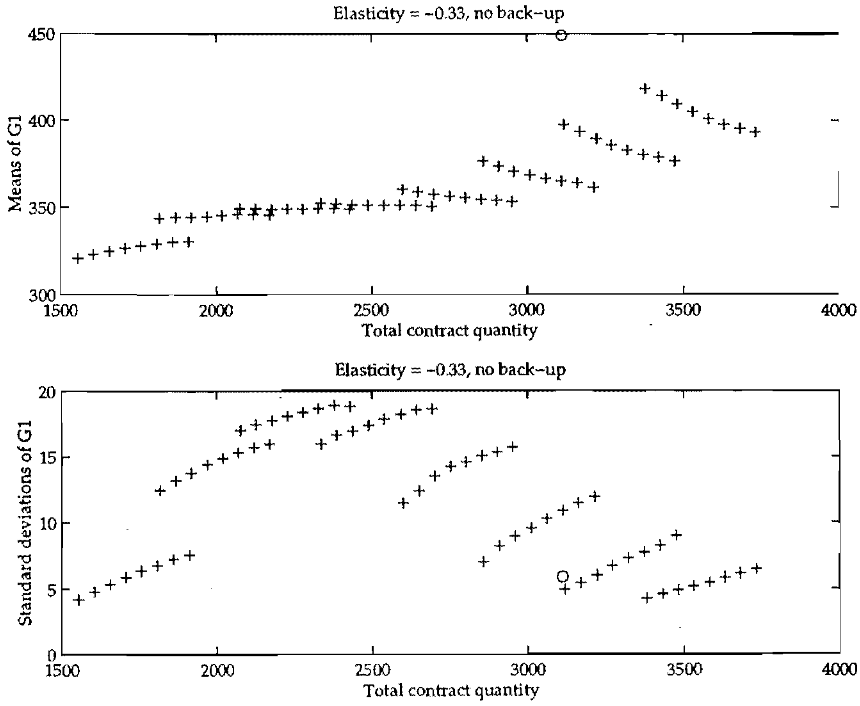


Figure A.2: Means and standard deviations of total generation, Firm One. Elasticity = -0.33, no back-up.

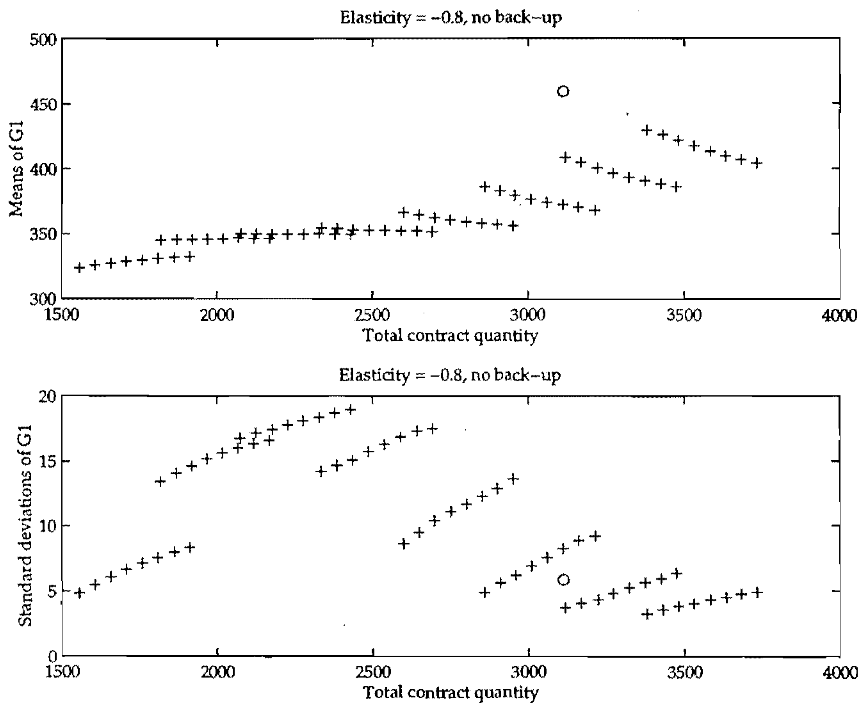


Figure A.3: Means and standard deviations of total generation, Firm One. Elasticity = -0.8, no back-up.

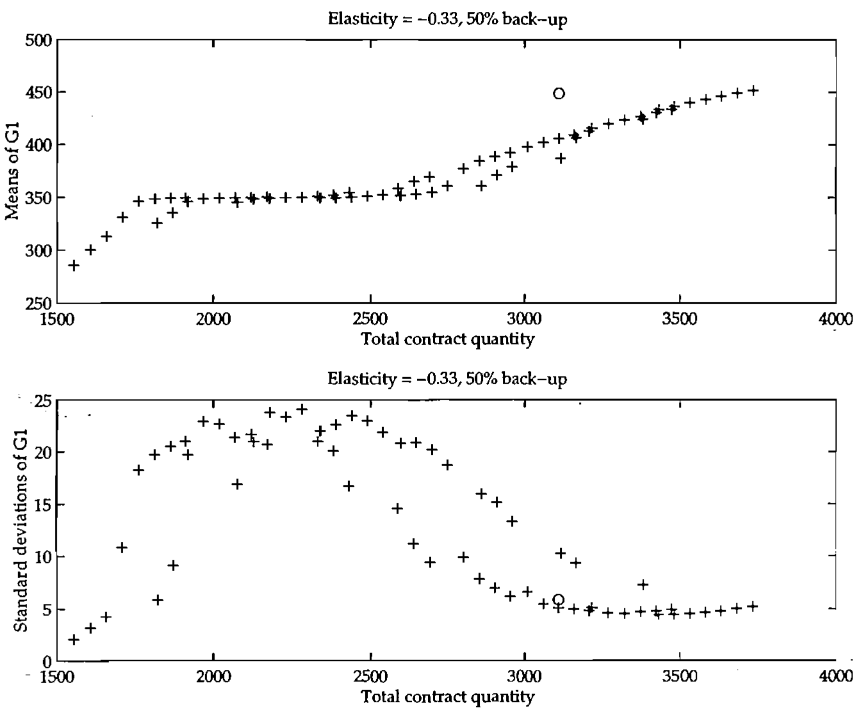


Figure A.4: Means and standard deviations of total generation, Firm One. Elasticity = -0.33, 50% back-up.

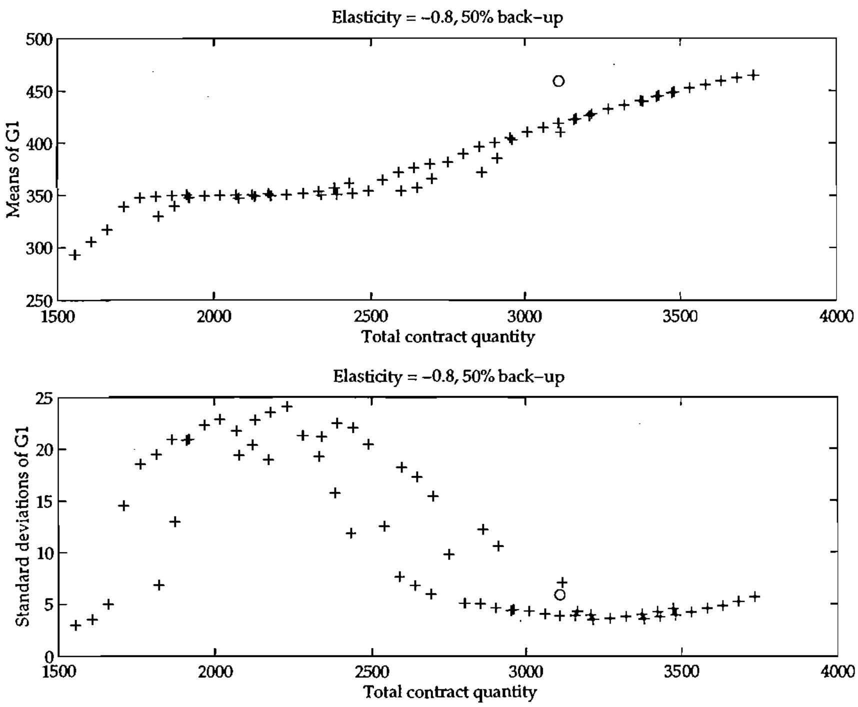


Figure A.5: Means and standard deviations of total generation, Firm One. Elasticity = -0.8, 50% back-up.

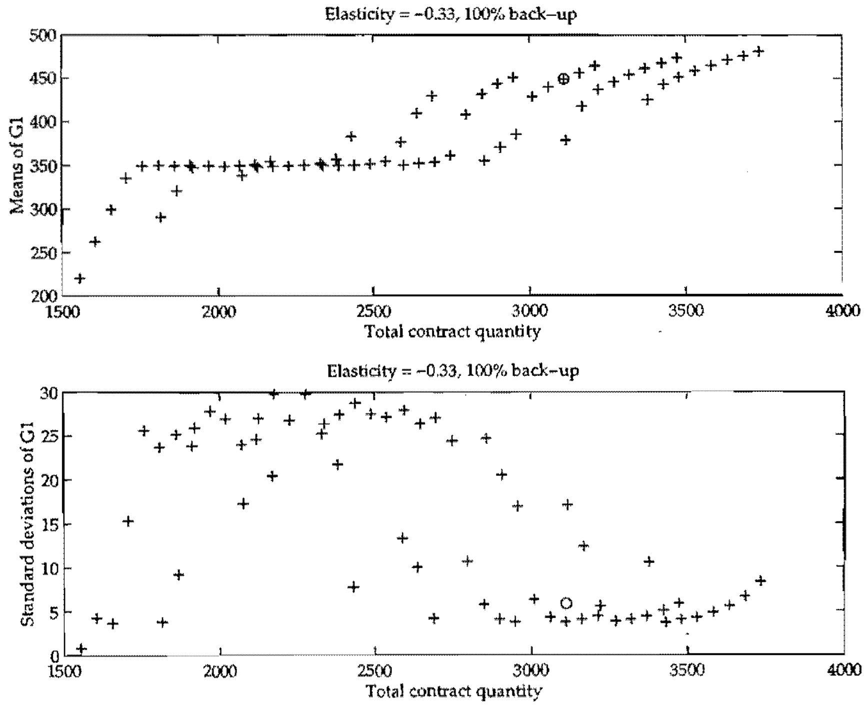


Figure A.6: Means and standard deviations of total generation, Firm One. Elasticity = -0.33, 100% back-up.

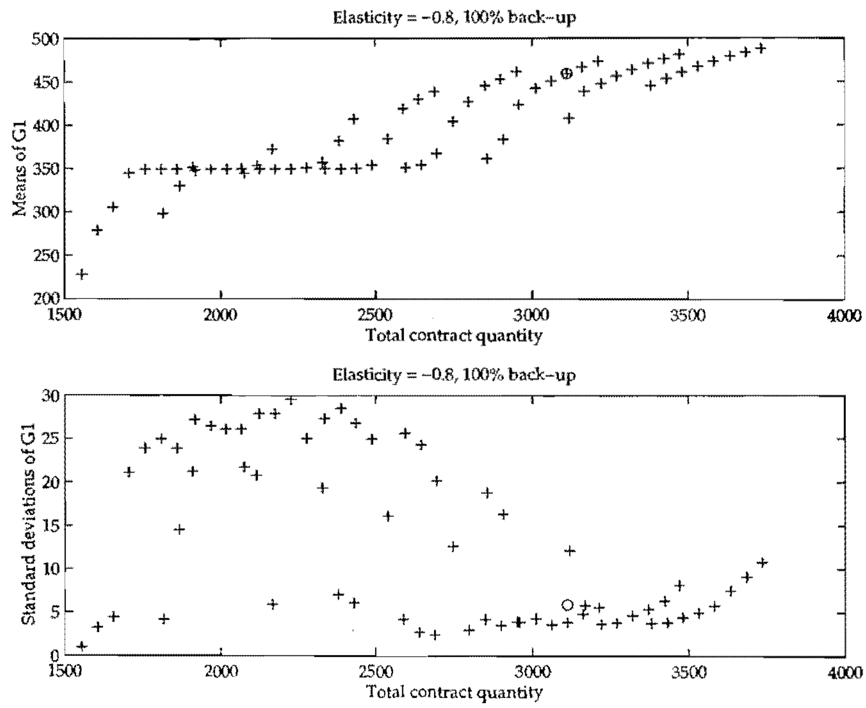


Figure A.7: Means and standard deviations of total generation, Firm One. Elasticity = -0.8, 100% back-up.

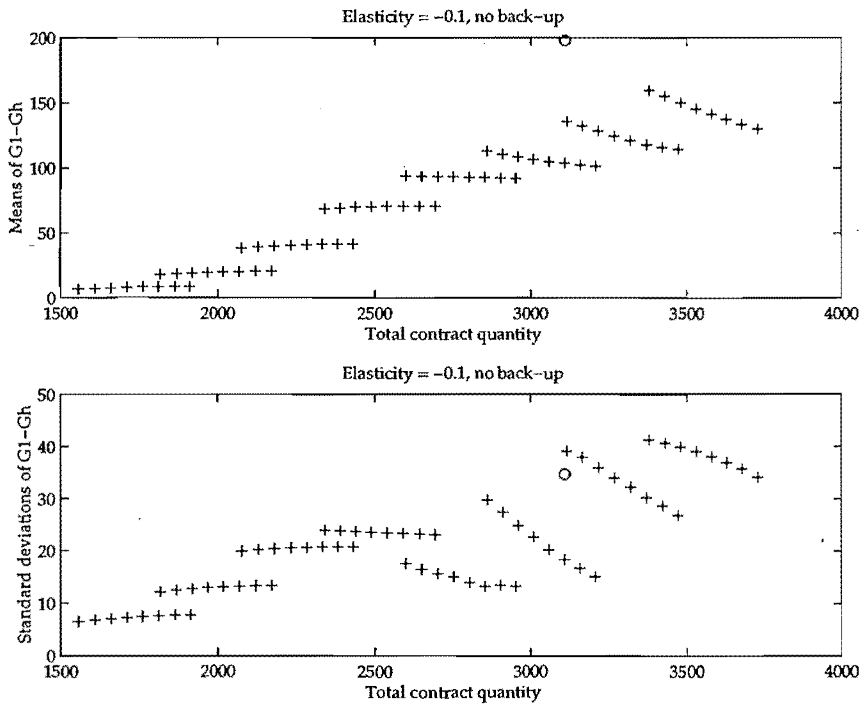


Figure A.8: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.1, no back-up.

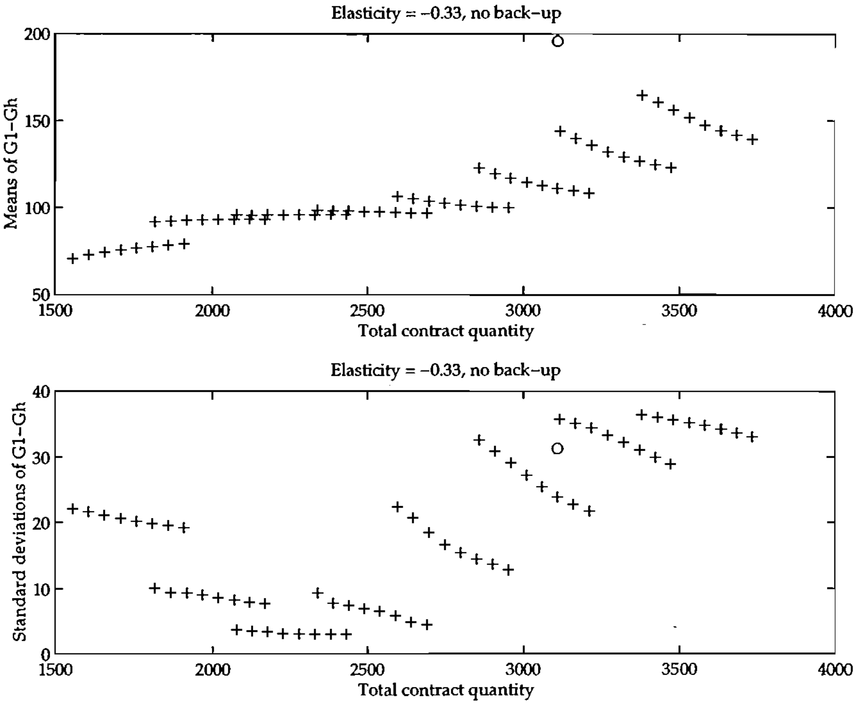


Figure A.9: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.33, no back-up.

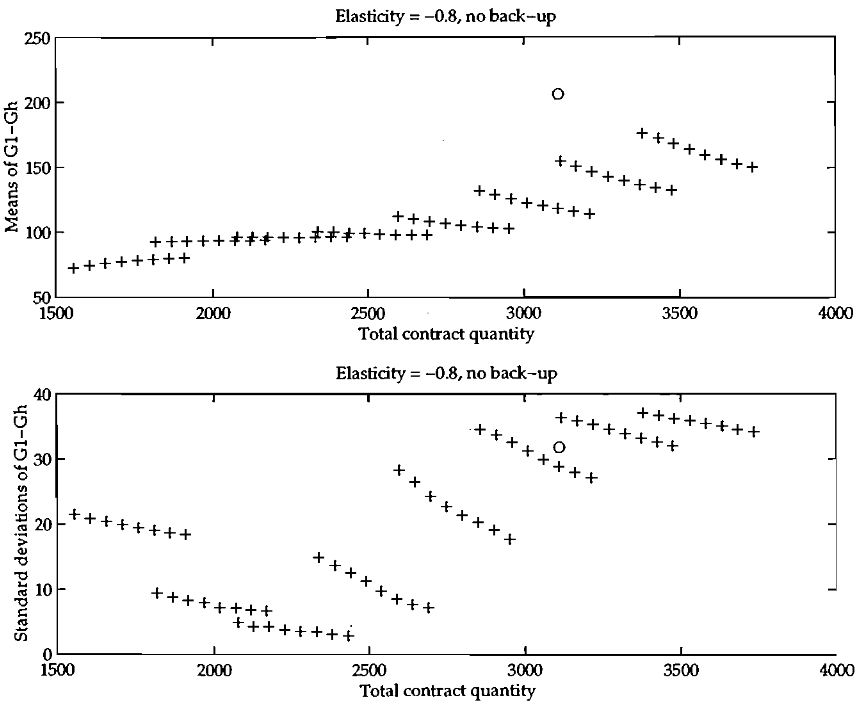


Figure A.10: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.8, no back-up.

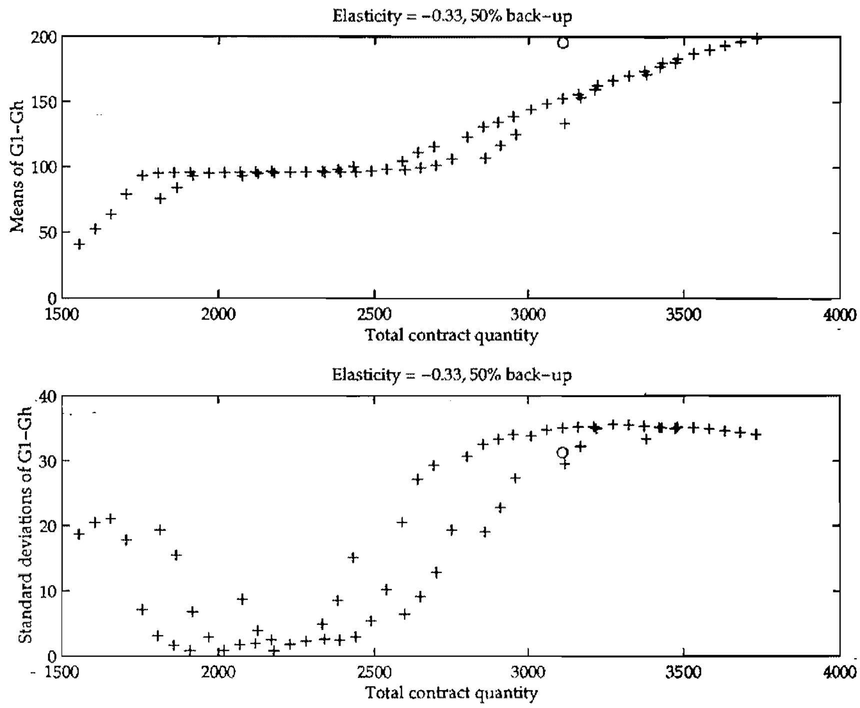


Figure A.11: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.33, 50% back-up.

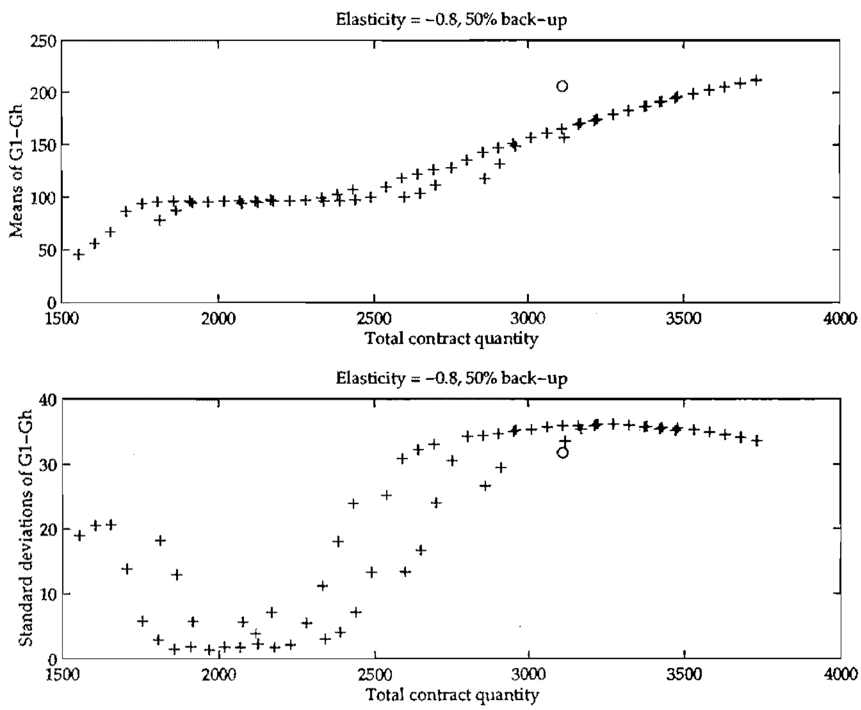


Figure A.12: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.8, 50% back-up.

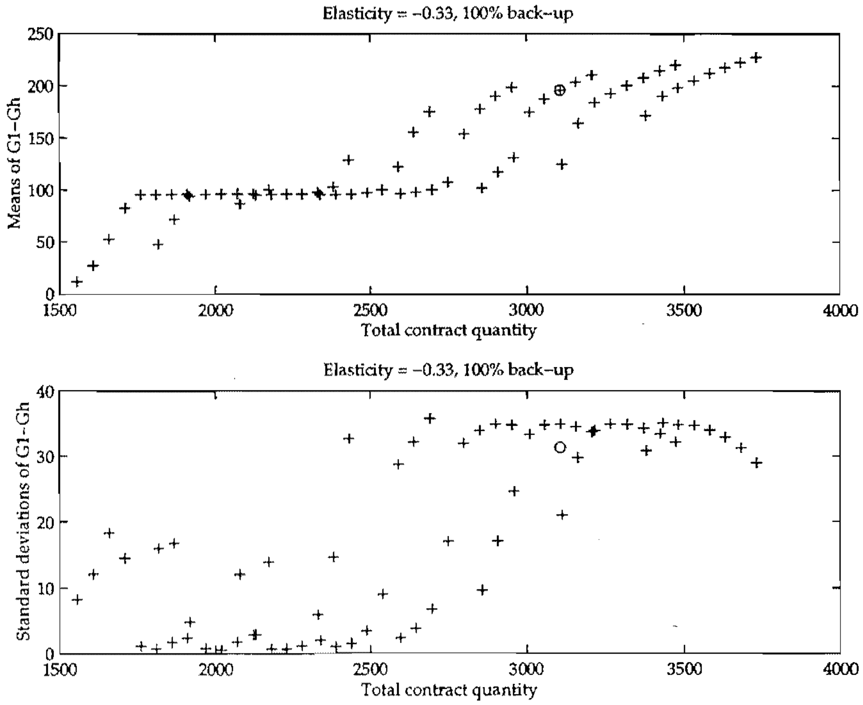


Figure A.13: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.33, 100% back-up.

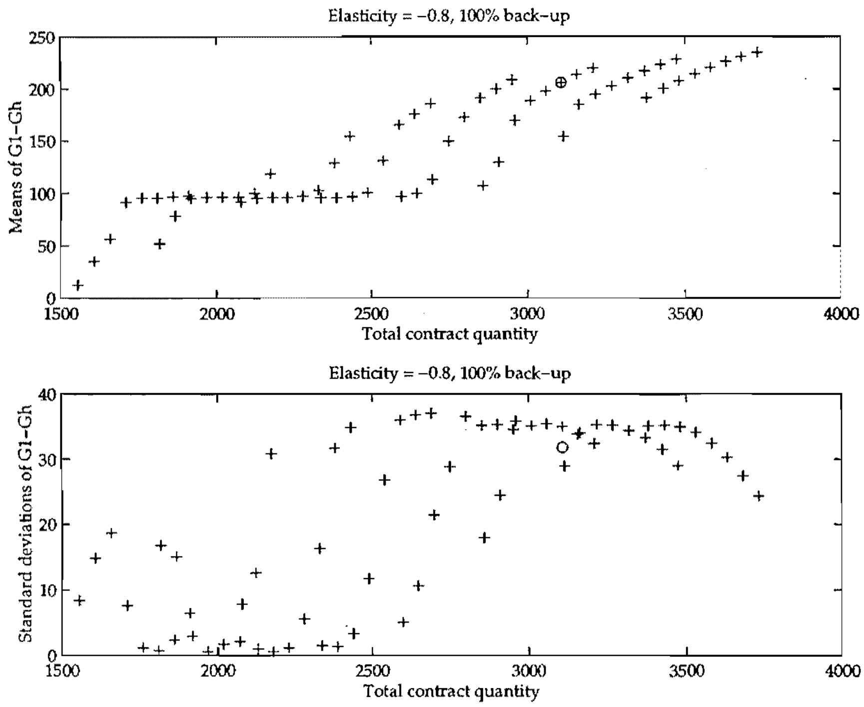


Figure A.14: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.8, 100% back-up.

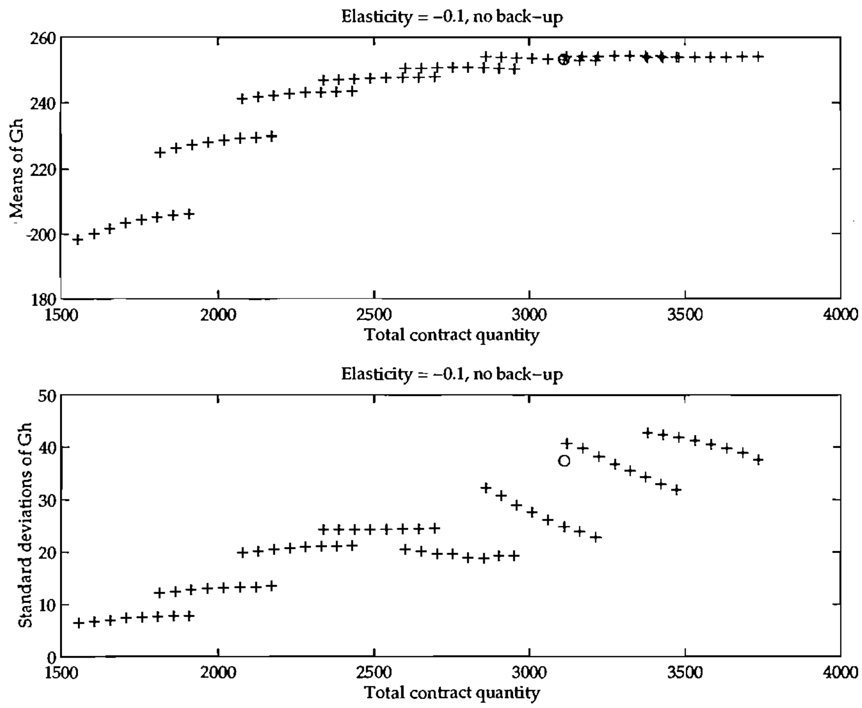


Figure A.15: Means and standard deviations of hydro generation. Elasticity = -0.1, no back-up.

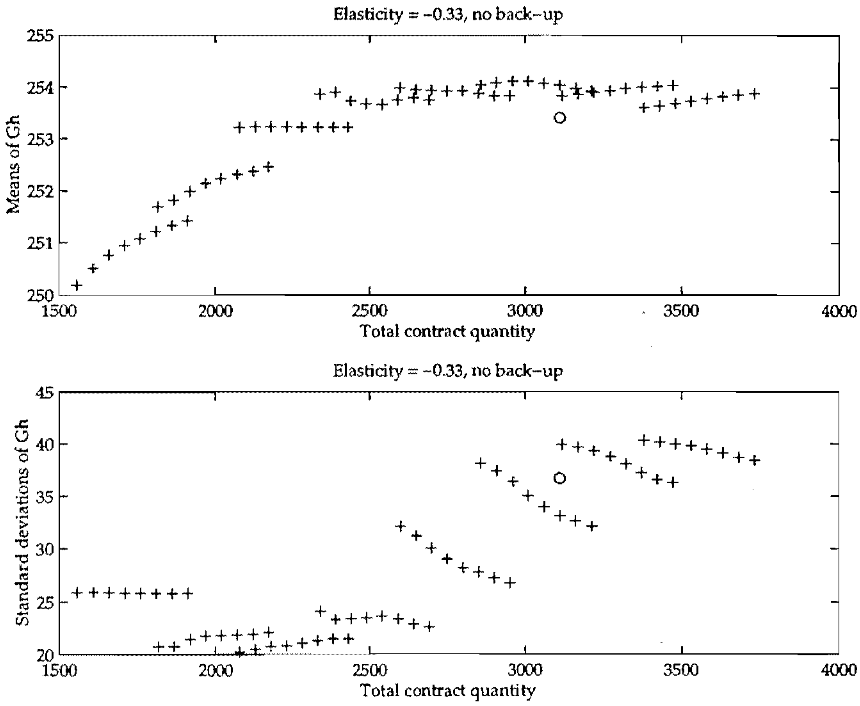


Figure A.16: Means and standard deviations of hydro generation. Elasticity = -0.33, no back-up.

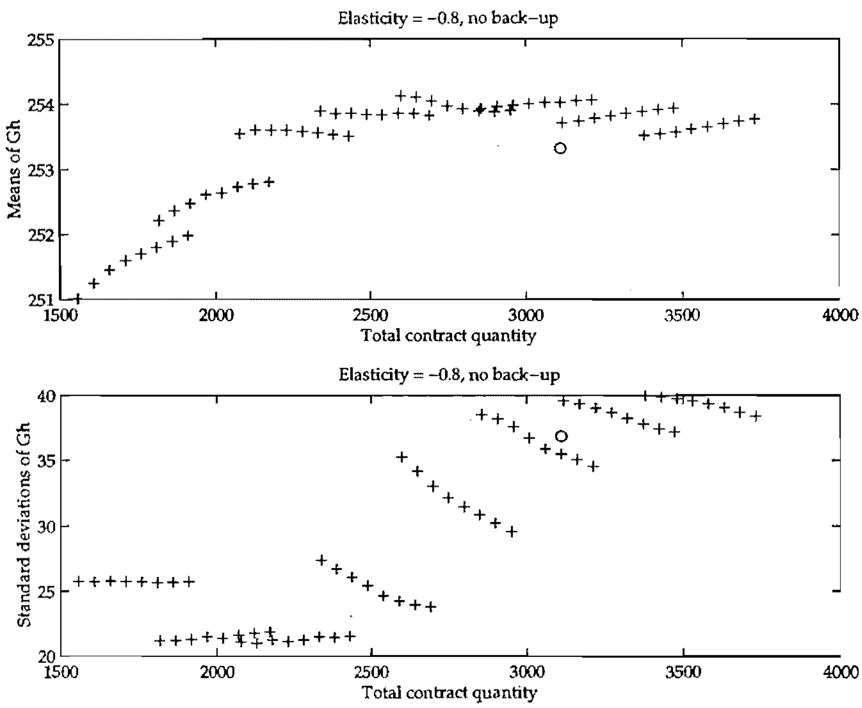


Figure A.17: Means and standard deviations of hydro generation. Elasticity = -0.8, no back-up.

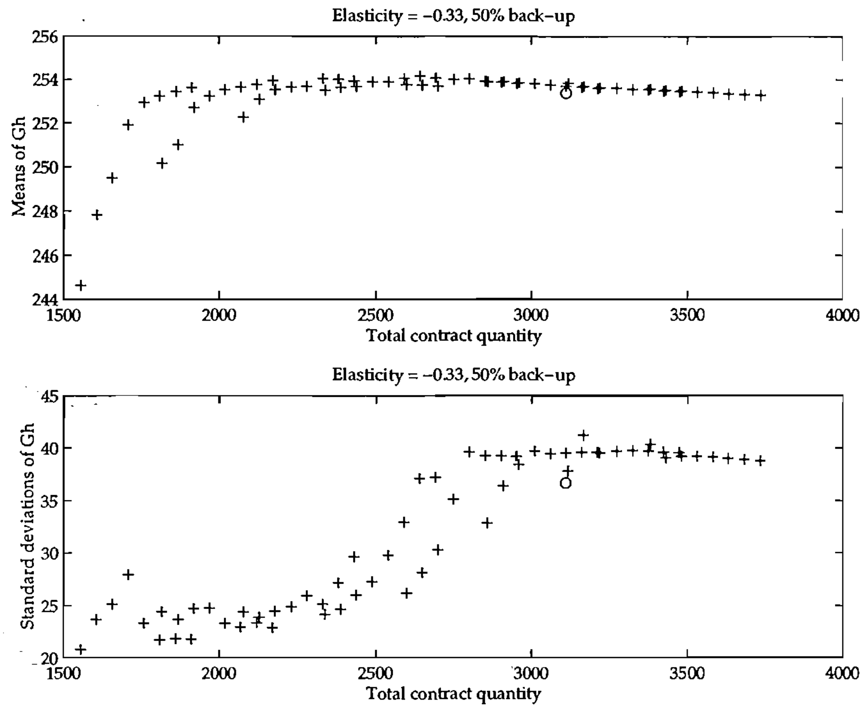


Figure A.18: Means and standard deviations of hydro generation. Elasticity = -0.33, 50% back-up.

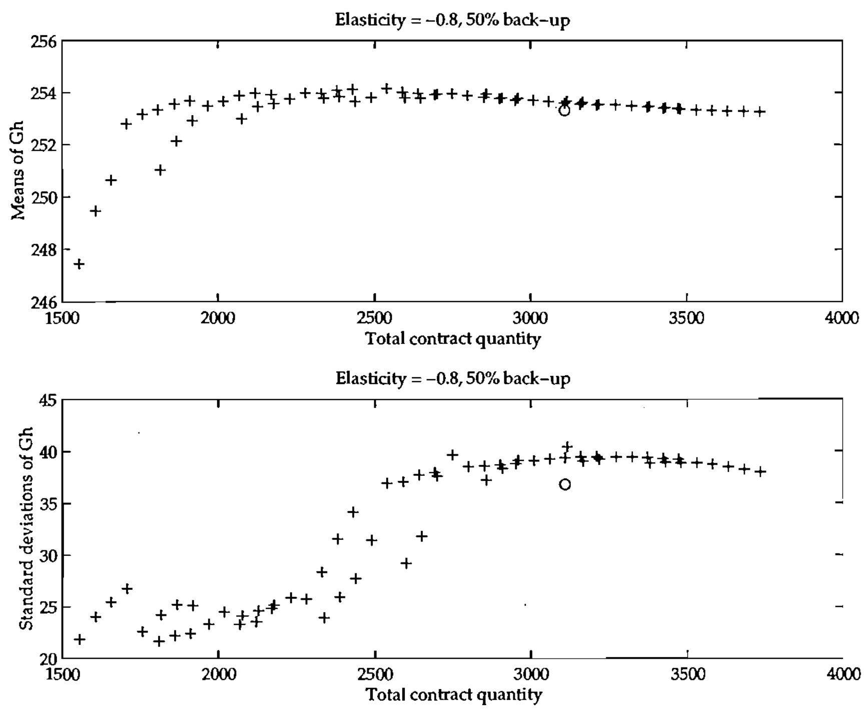


Figure A.19: Means and standard deviations of hydro generation. Elasticity = -0.8, 50% back-up.

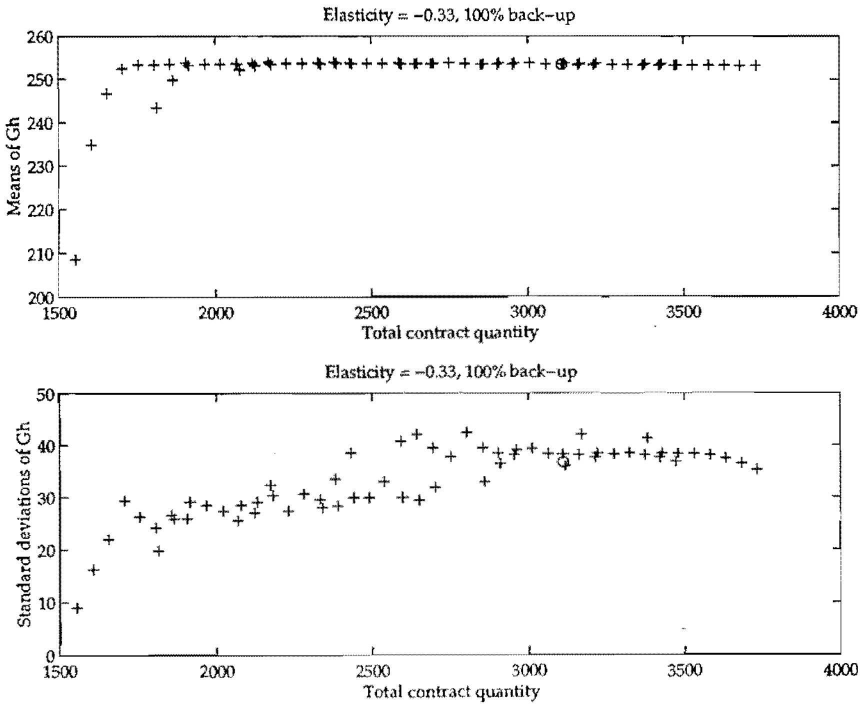


Figure A.20: Means and standard deviations of hydro generation. Elasticity = -0.33, 100% back-up.

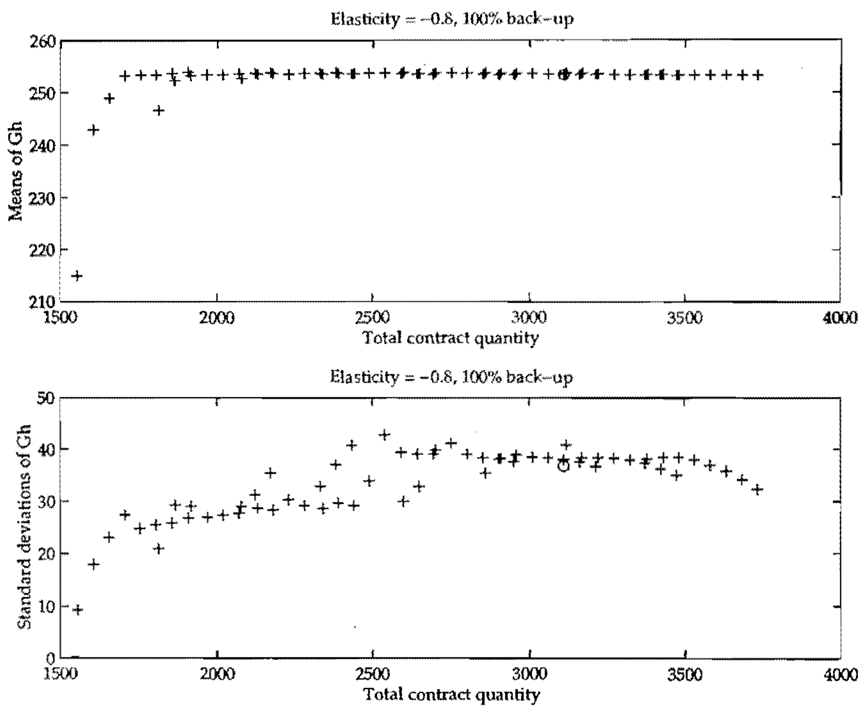
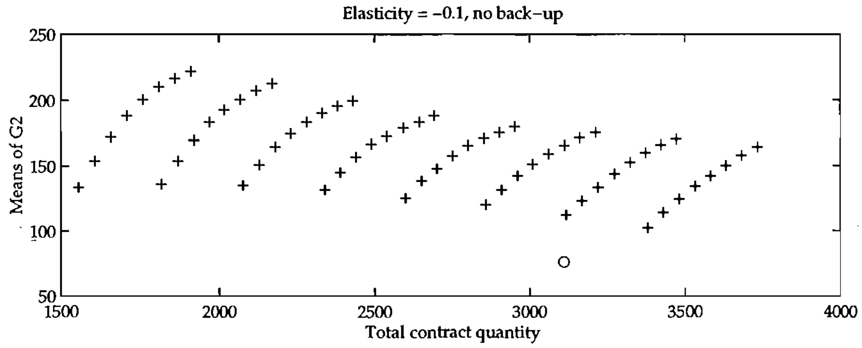


Figure A.21: Means and standard deviations of hydro generation. Elasticity = -0.8, 100% back-up.



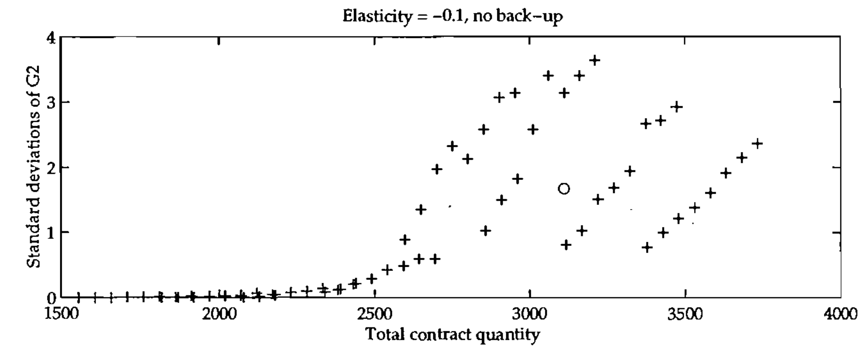
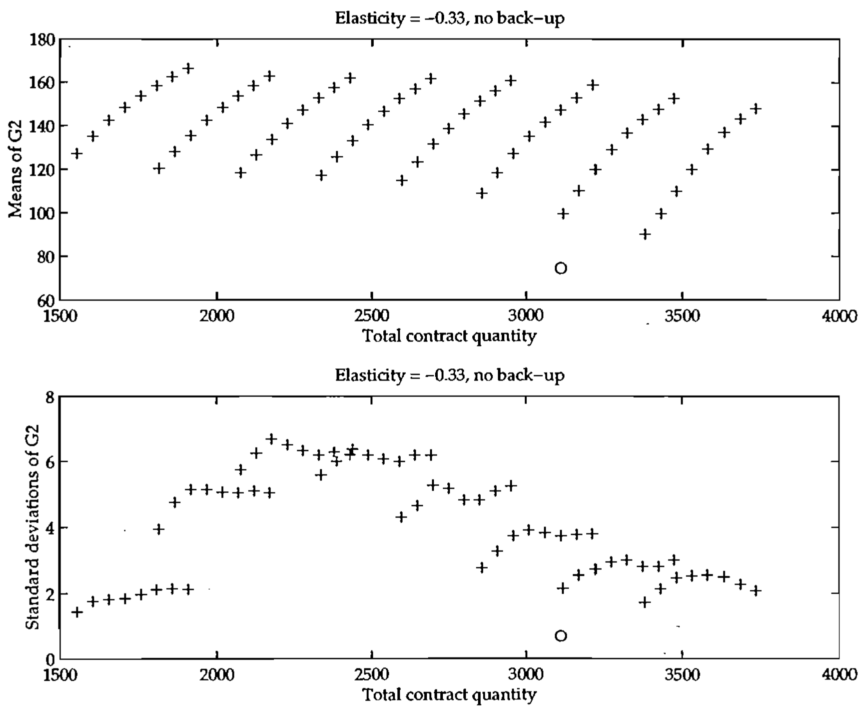


Figure A.22: Means and standard deviations of generation, Firm Two. Elasticity = -0.1, no back-up.



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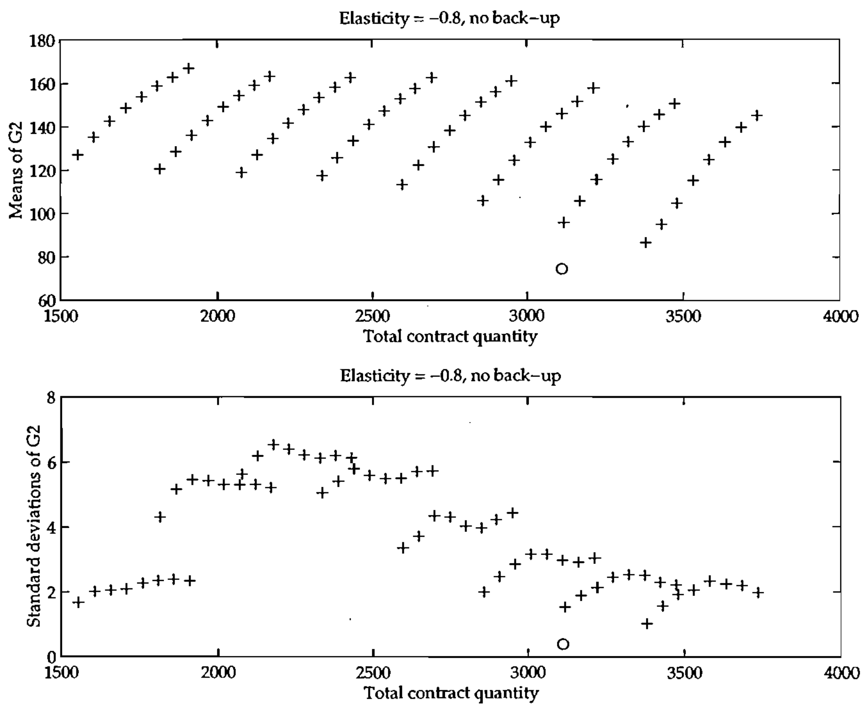

Figure A.24: Means and standard deviations of generation, Firm Two. Elasticity = -0.8, no back-up.

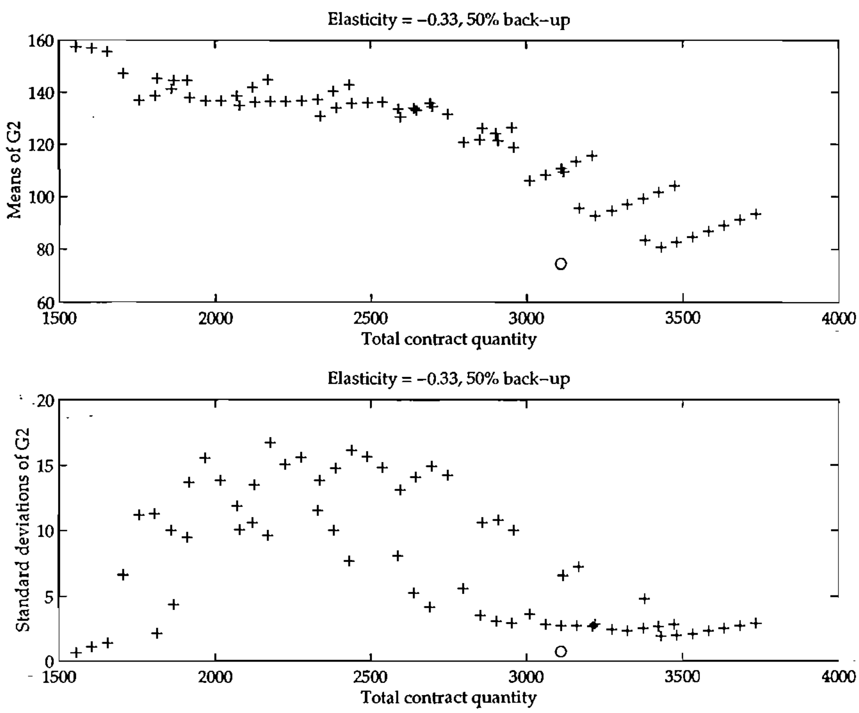

Figure A.25: Means and standard deviations of generation, Firm Two. Elasticity = -0.33, 50% back-up.

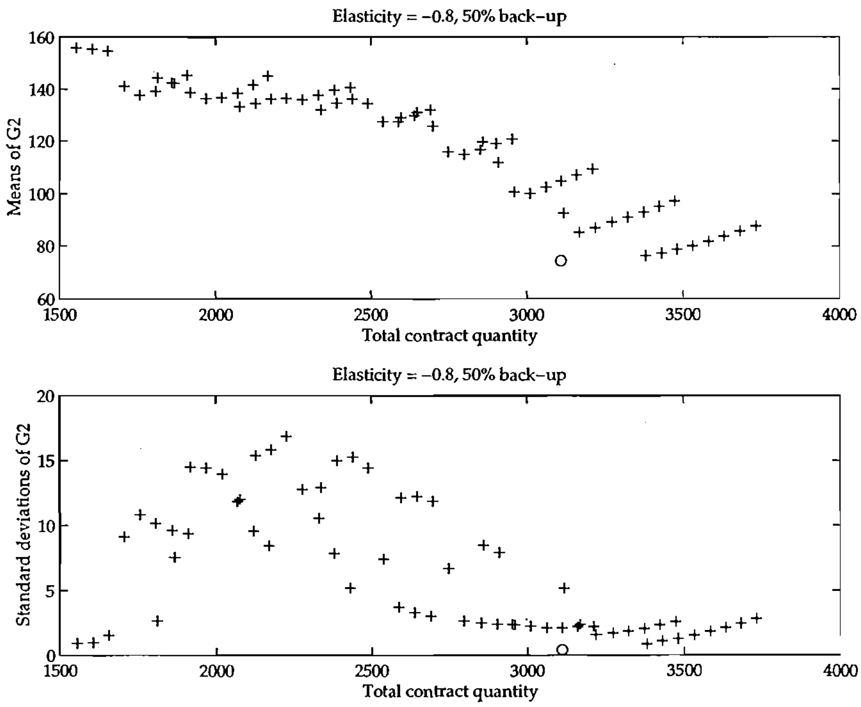

Figure A.26: Means and standard deviations of generation, Firm Two. Elasticity = -0.8, 50% back-up.

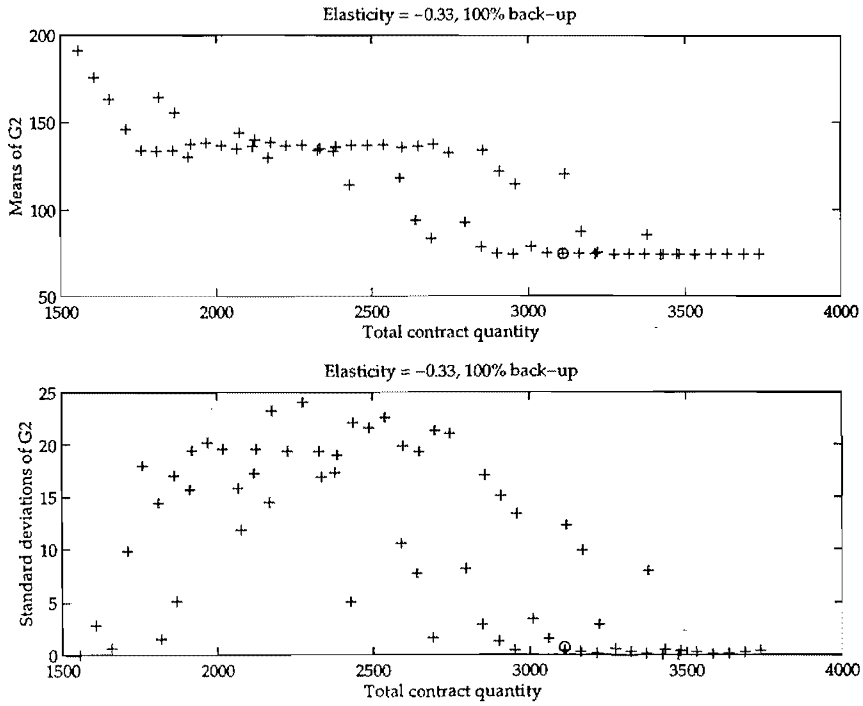

Figure A.27: Means and standard deviations of generation, Firm Two. Elasticity = -0.33, 100% back-up.

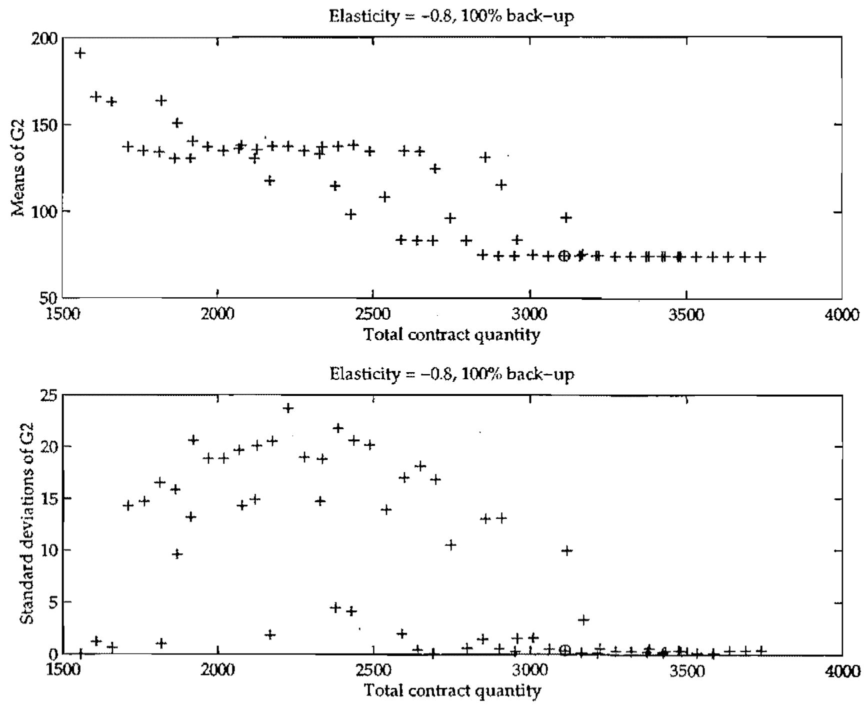

Figure A.28: Means and standard deviations of generation, Firm Two. Elasticity = -0.8, 100% back-up.

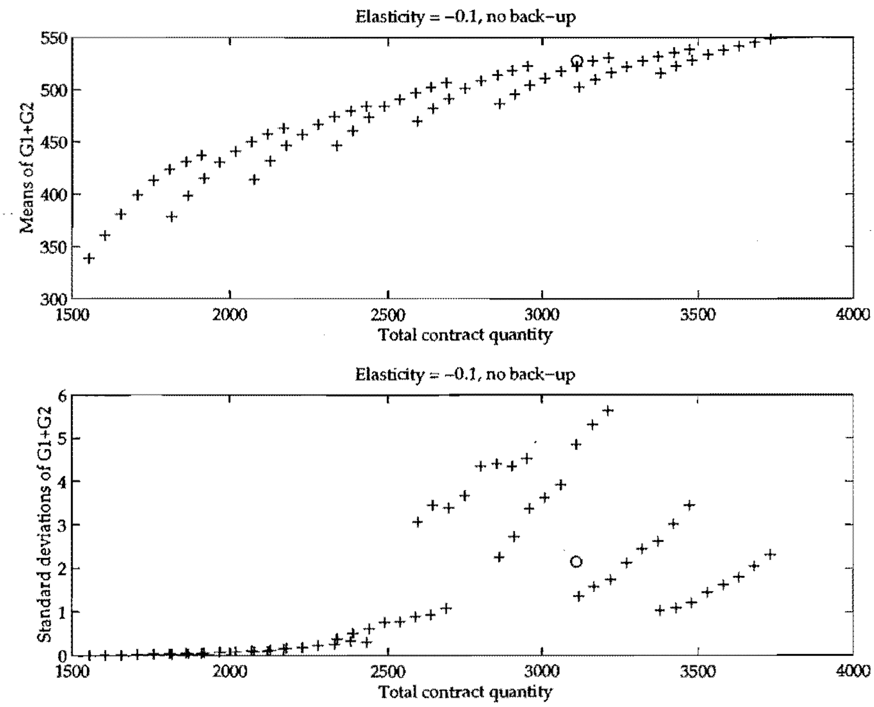

Figure A.29: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.1, no back-up.

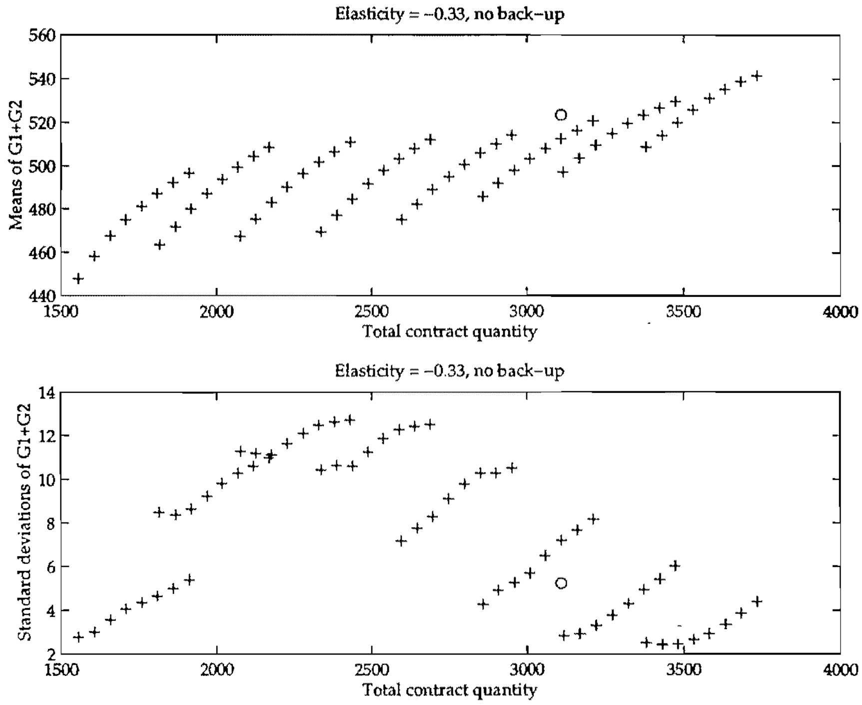

Figure A.30: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.33, no back-up.

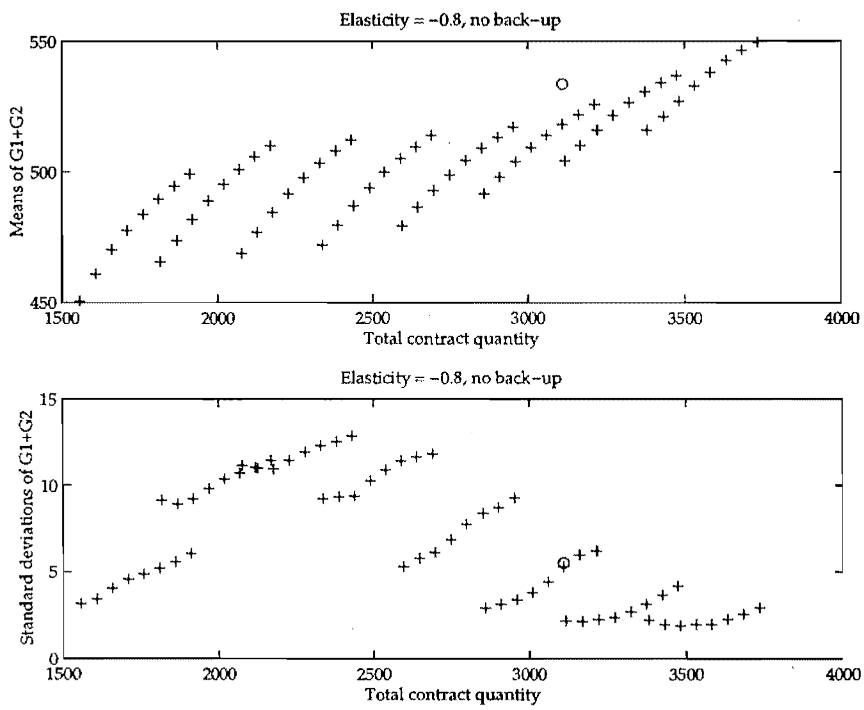

Figure A.31: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.8, no back-up.

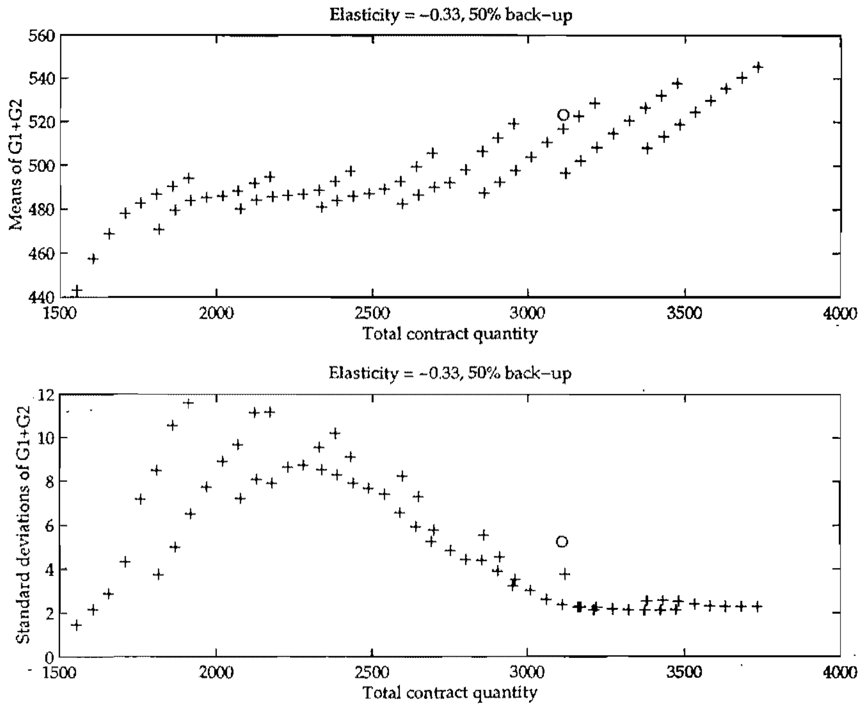

Figure A.32: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.33, 50% back-up.

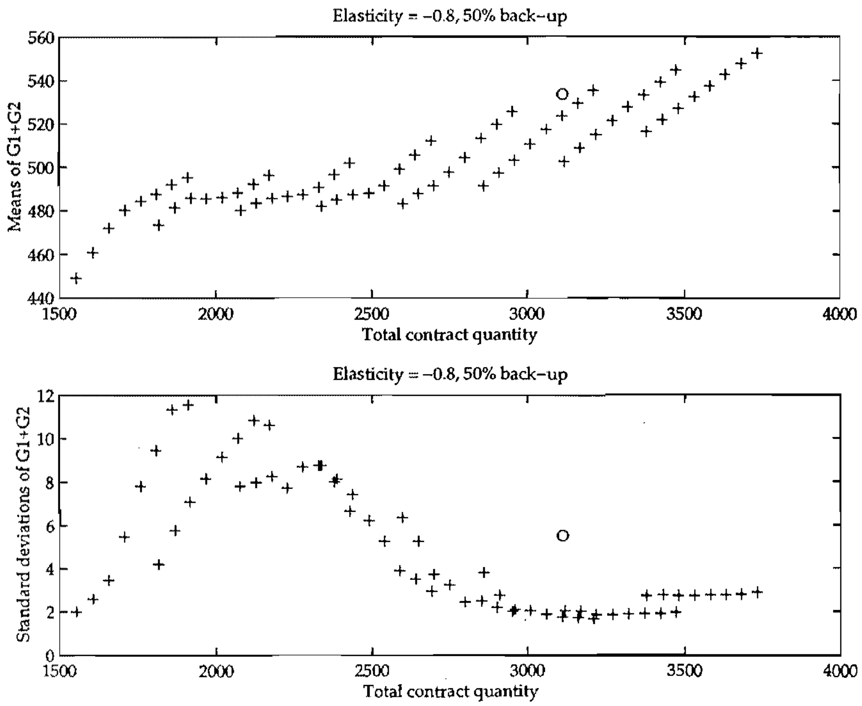

Figure A.33: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.8, 50% back-up.

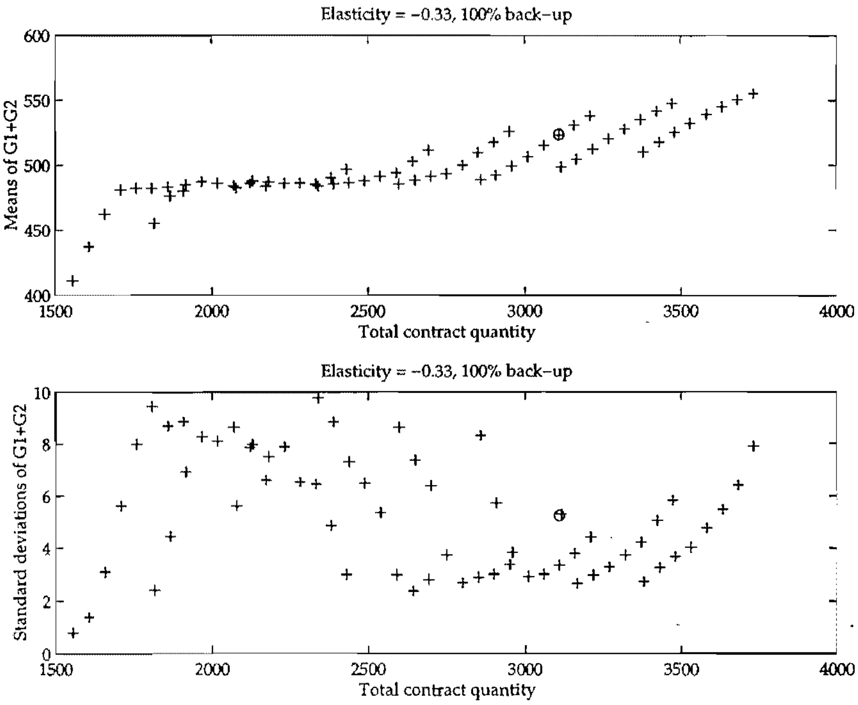

Figure A.34: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.33, 100% back-up.

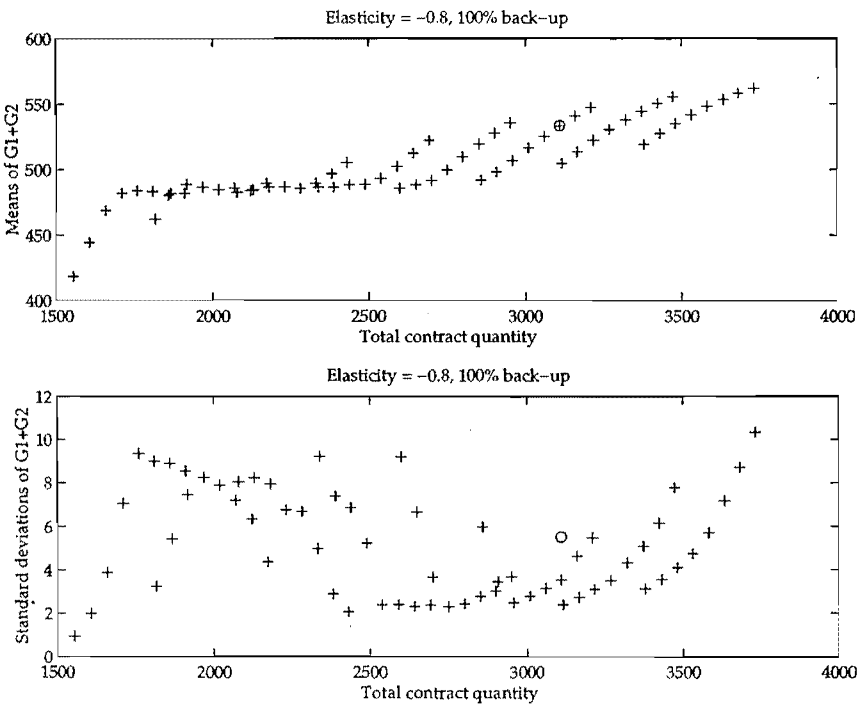

Figure A.35: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.8, 100% back-up.

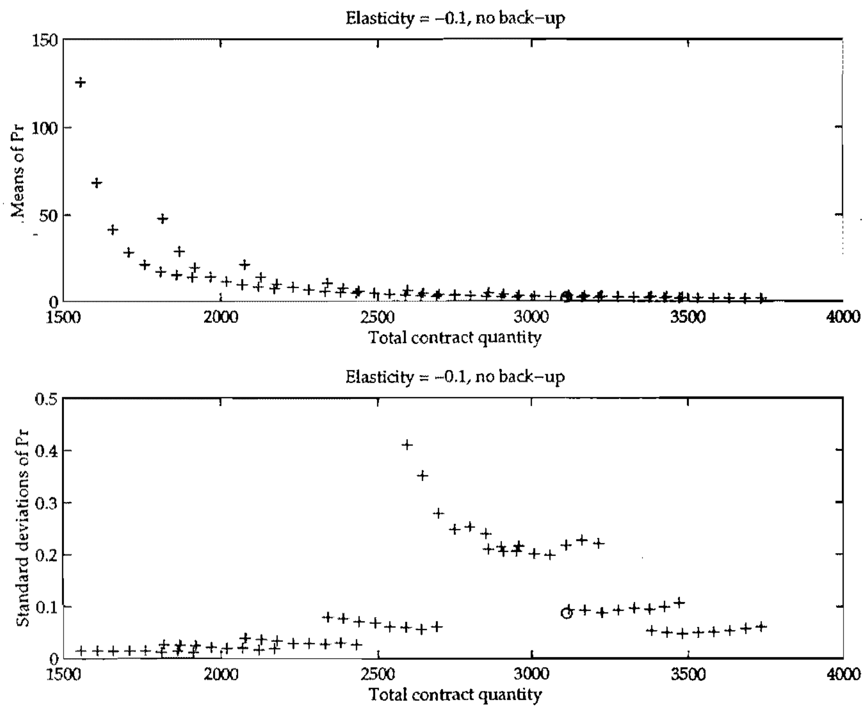

Figure A.36: Means and standard deviations of energy spot price. Elasticity = -0.1, no back-up.

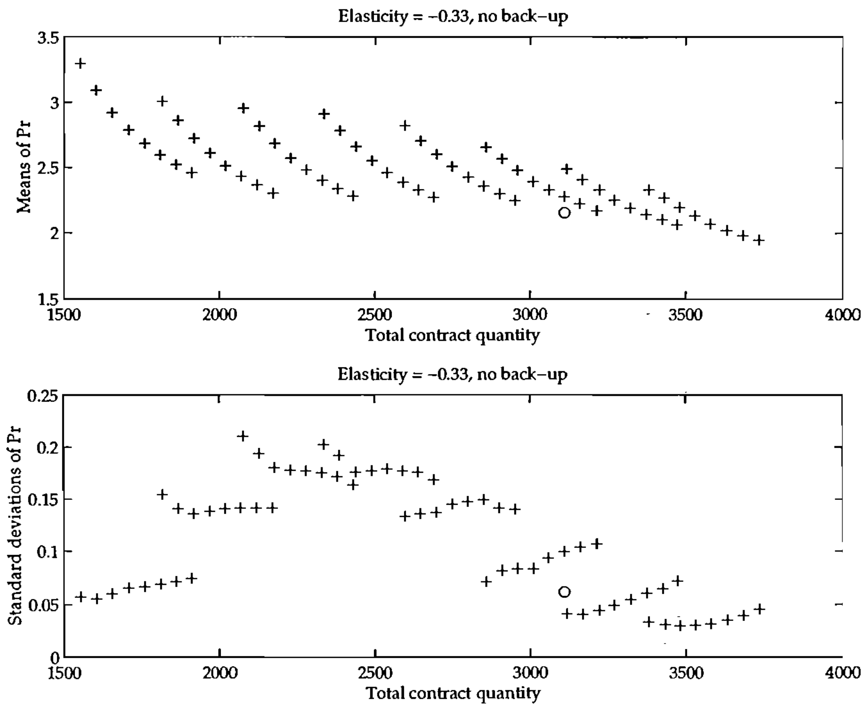

Figure A.37: Means and standard deviations of energy spot price. Elasticity = -0.33, no back-up.

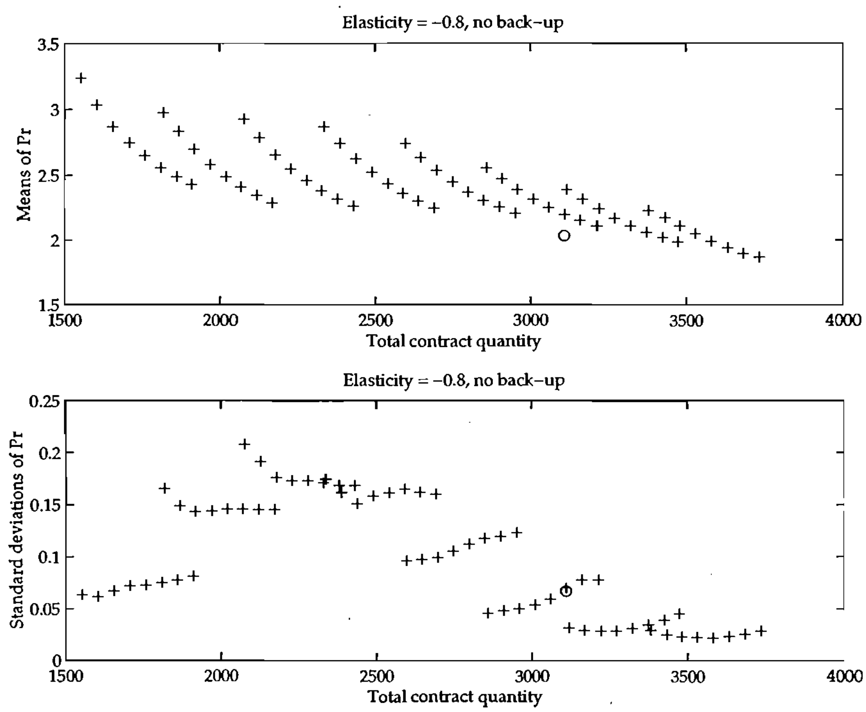

Figure A.38: Means and standard deviations of energy spot price. Elasticity = -0.8, no back-up.

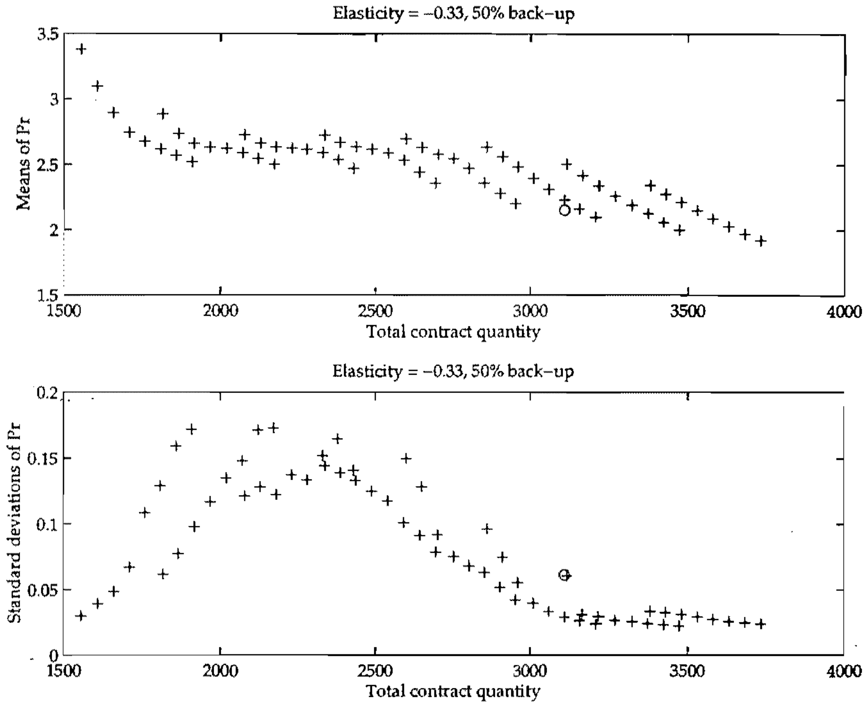

Figure A.39: Means and standard deviations of energy spot price. Elasticity = -0.33, 50% back-up.

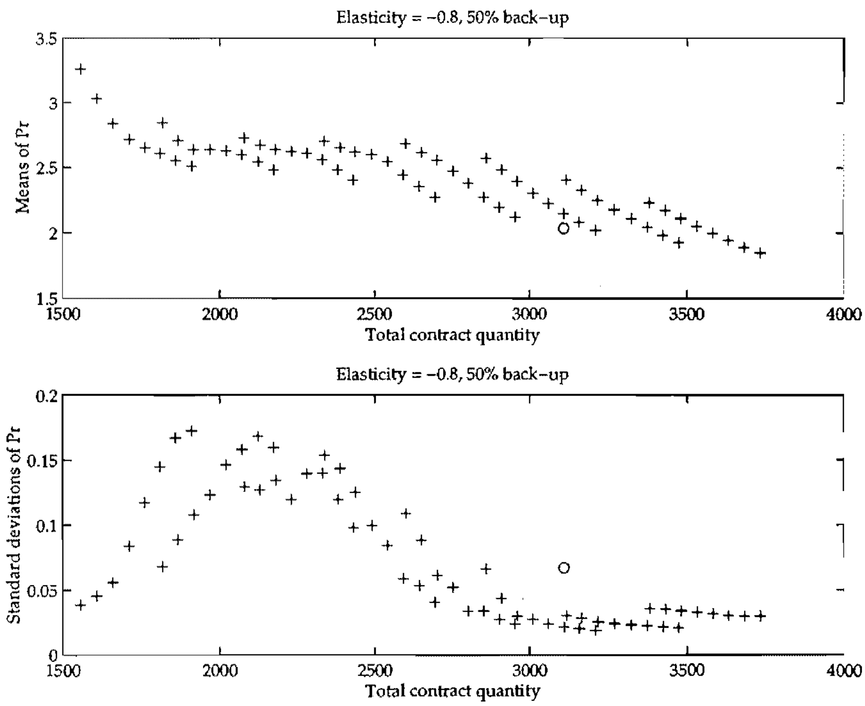

Figure A.40: Means and standard deviations of energy spot price. Elasticity = -0.8, 50% back-up.

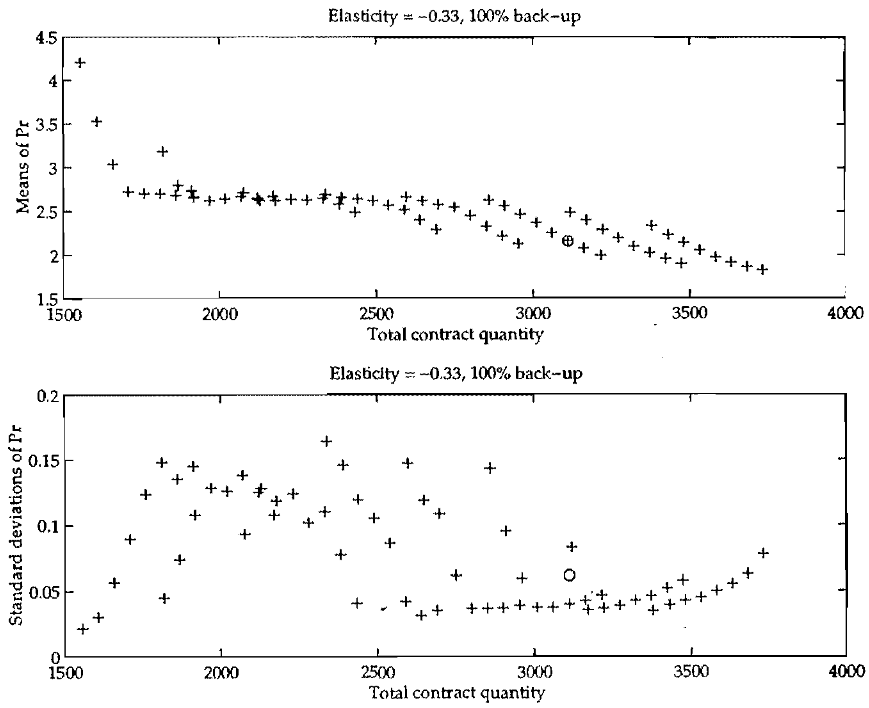

Figure A.41: Means and standard deviations of energy spot price. Elasticity = -0.33, 100% back-up.

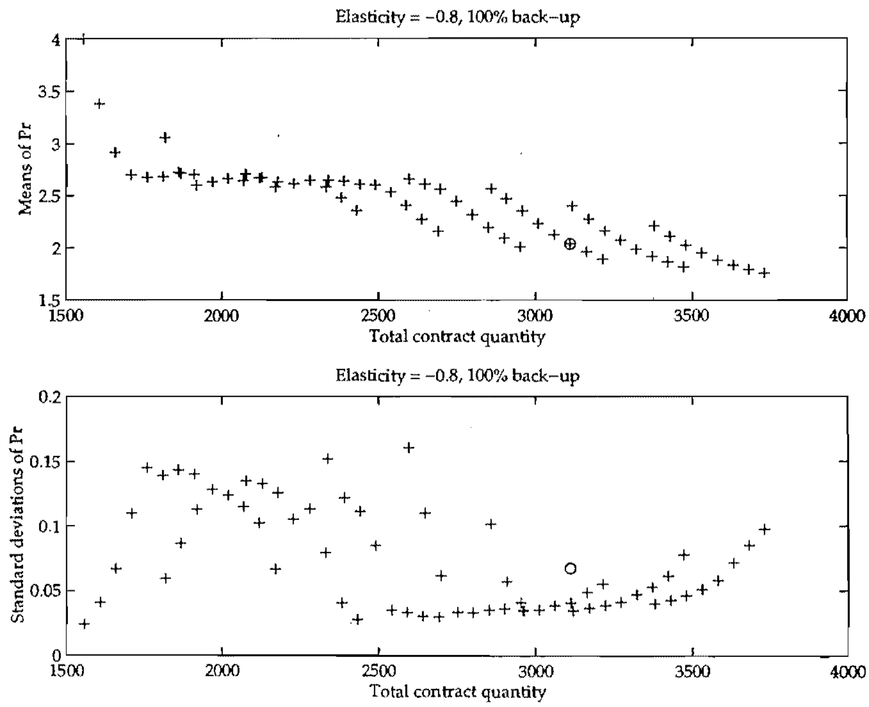

Figure A.42: Means and standard deviations of energy spot price. Elasticity = -0.8, 100% back-up.

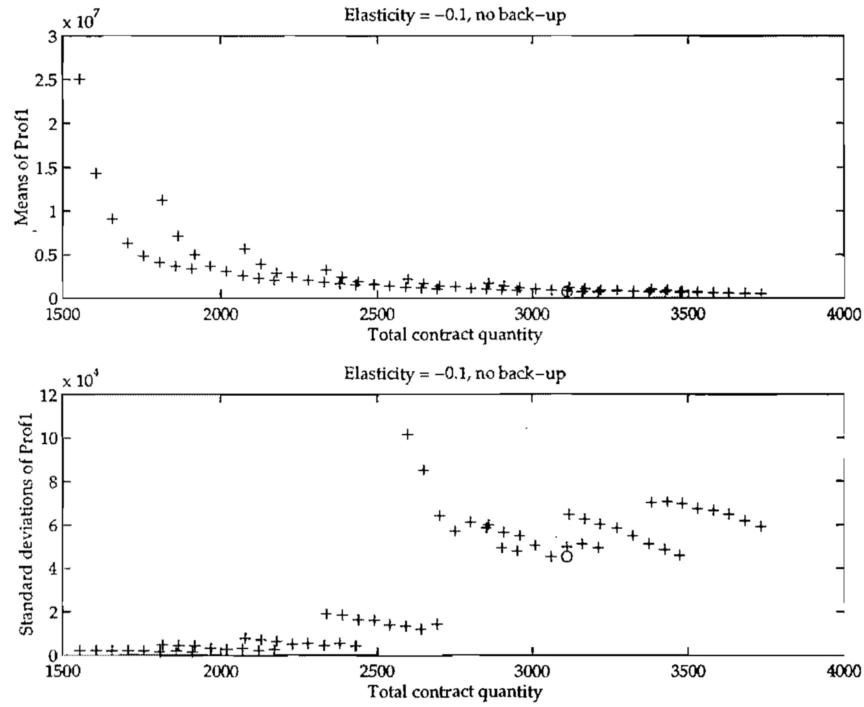

Figure A.43: Means and standard deviations of profit, Firm One. Elasticity = -0.1, no back-up.

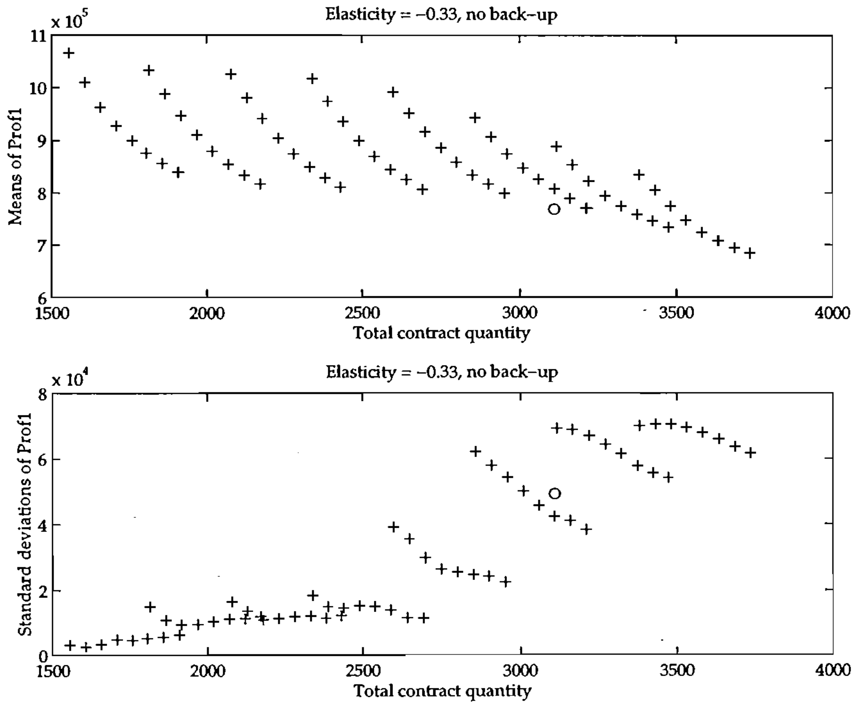

Figure A.44: Means and standard deviations of profit, Firm One. Elasticity = -0.33, no back-up.

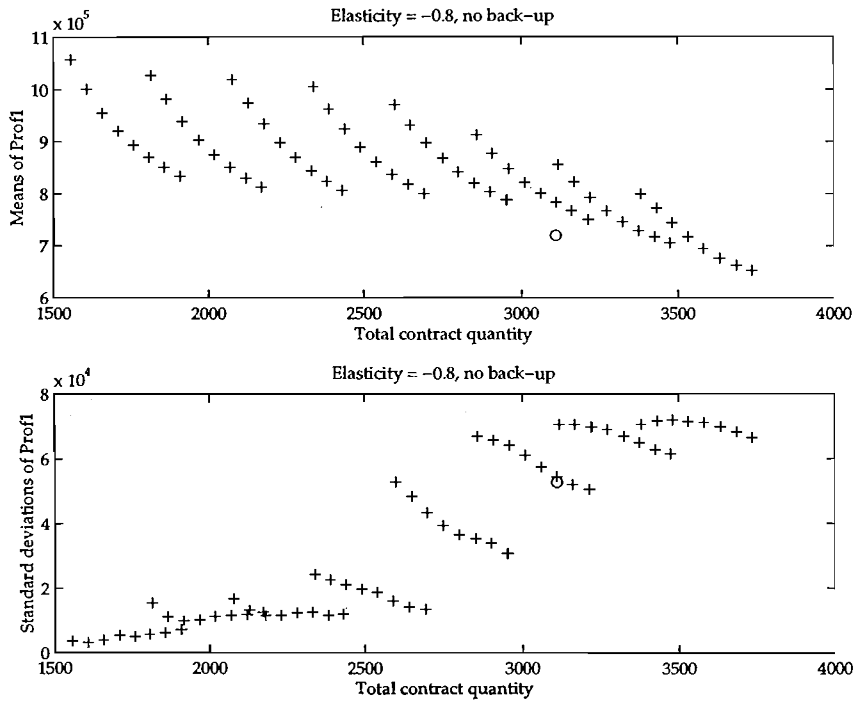

Figure A.45: Means and standard deviations of profit, Firm One. Elasticity = -0.8, no back-up.

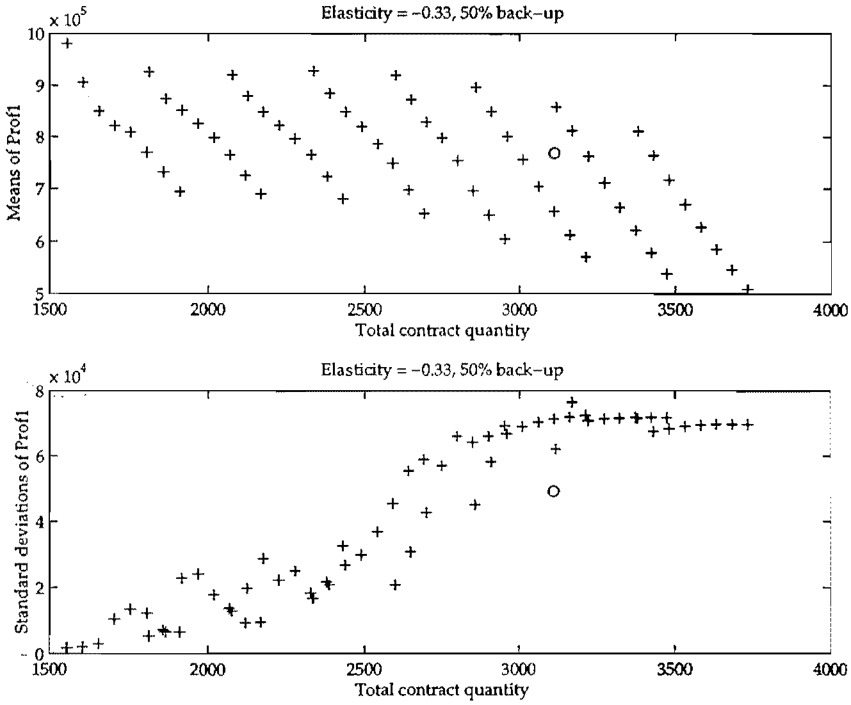

Figure A.46: Means and standard deviations of profit, Firm One. Elasticity = -0.33, 50% back-up.

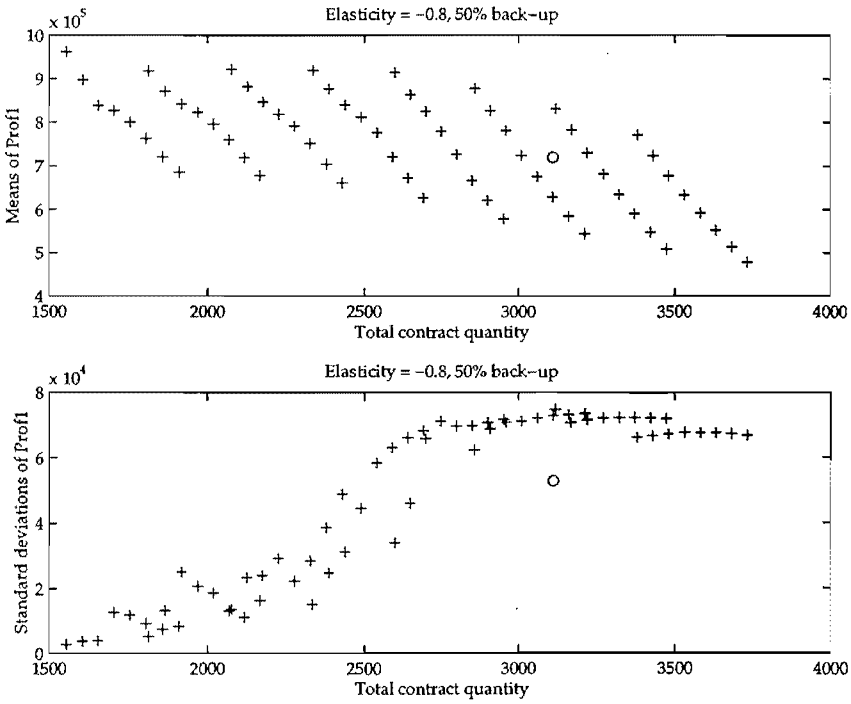

Figure A.47: Means and standard deviations of profit, Firm One. Elasticity = -0.8, 50% back-up.

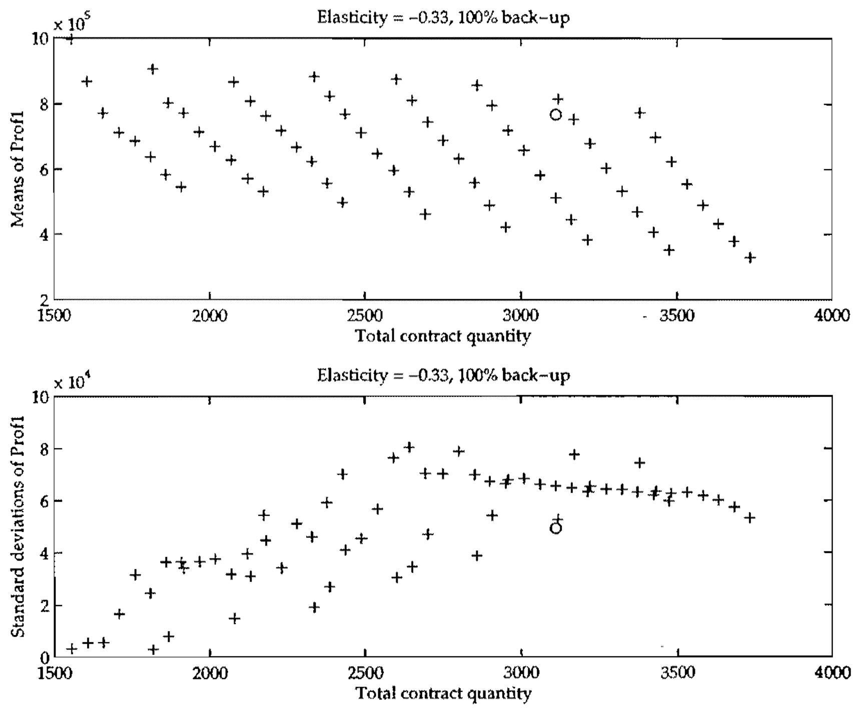

Figure A.48: Means and standard deviations of profit, Firm One. Elasticity = -0.33, 100% back-up.

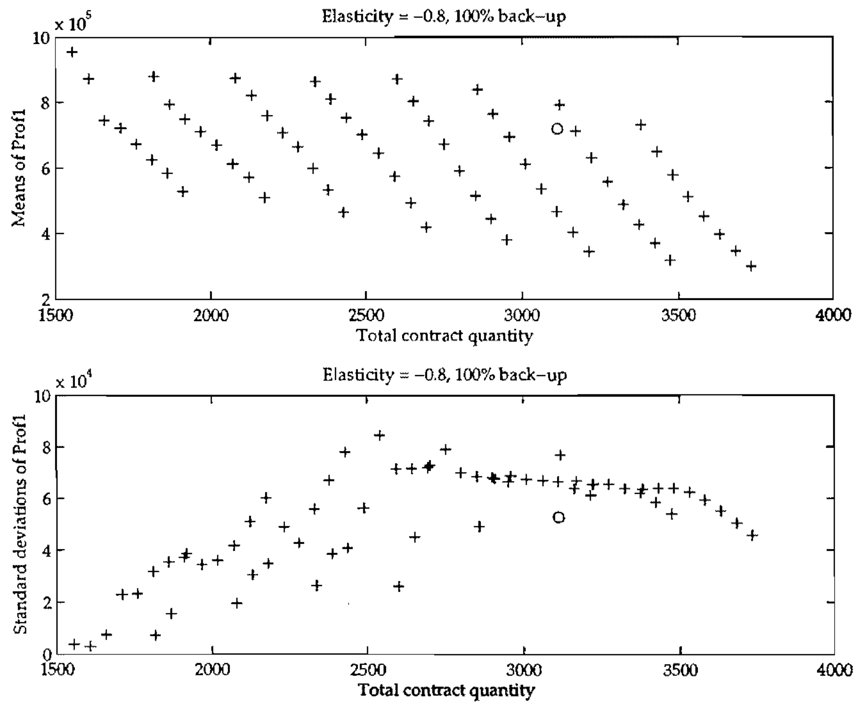

Figure A.49: Means and standard deviations of profit, Firm One. Elasticity = -0.8, 100% back-up.

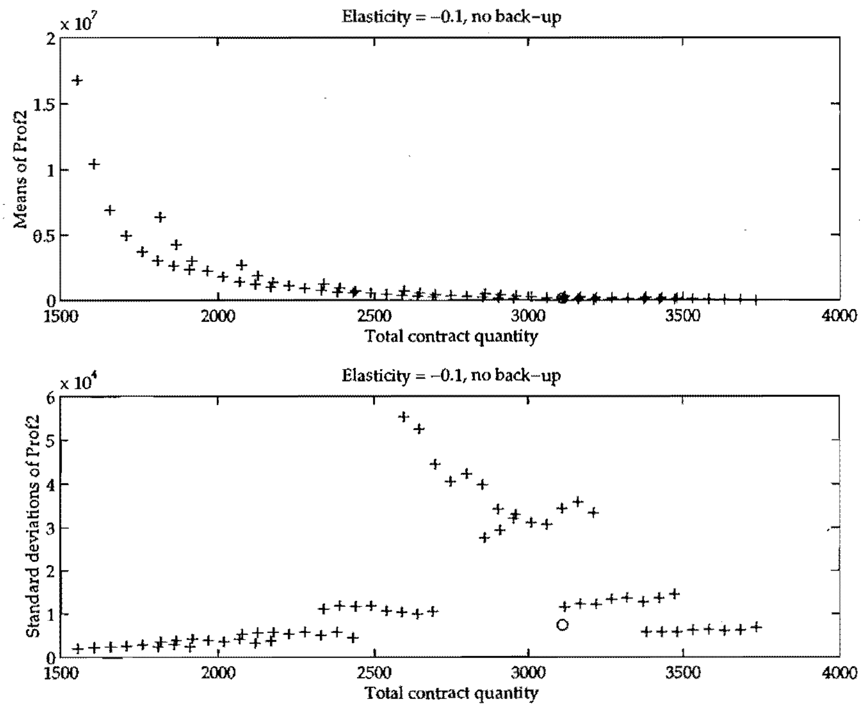

Figure A.50: Means and standard deviations of profit, Firm Two. Elasticity = -0.1, no back-up.

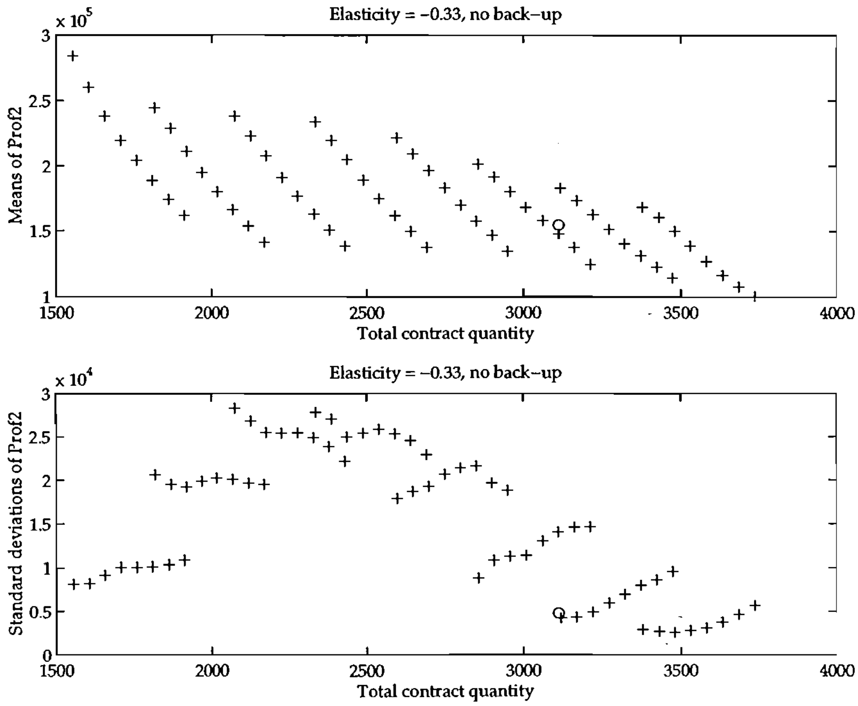

Figure A.51: Means and standard deviations of profit, Firm Two. Elasticity = -0.33, no back-up.

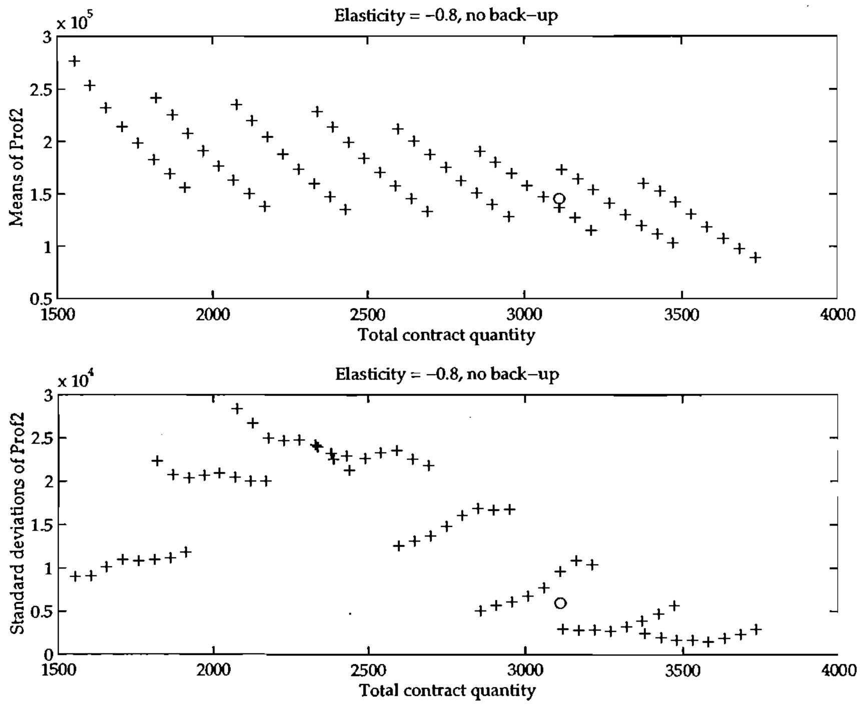

Figure A.52: Means and standard deviations of profit, Firm Two. Elasticity = -0.8, no back-up.

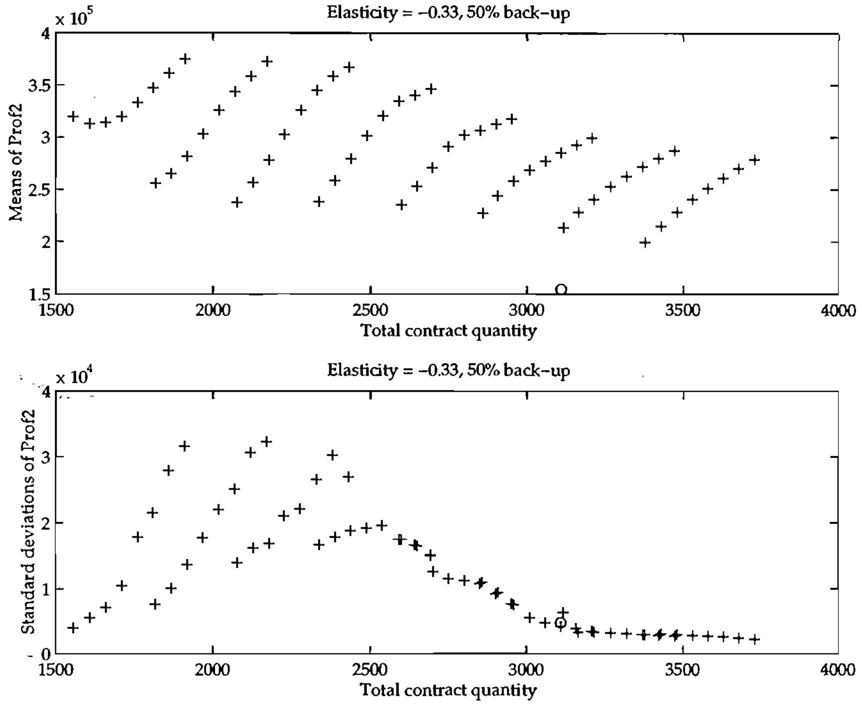

Figure A.53: Means and standard deviations of profit, Firm Two. Elasticity = -0.33, 50% back-up.

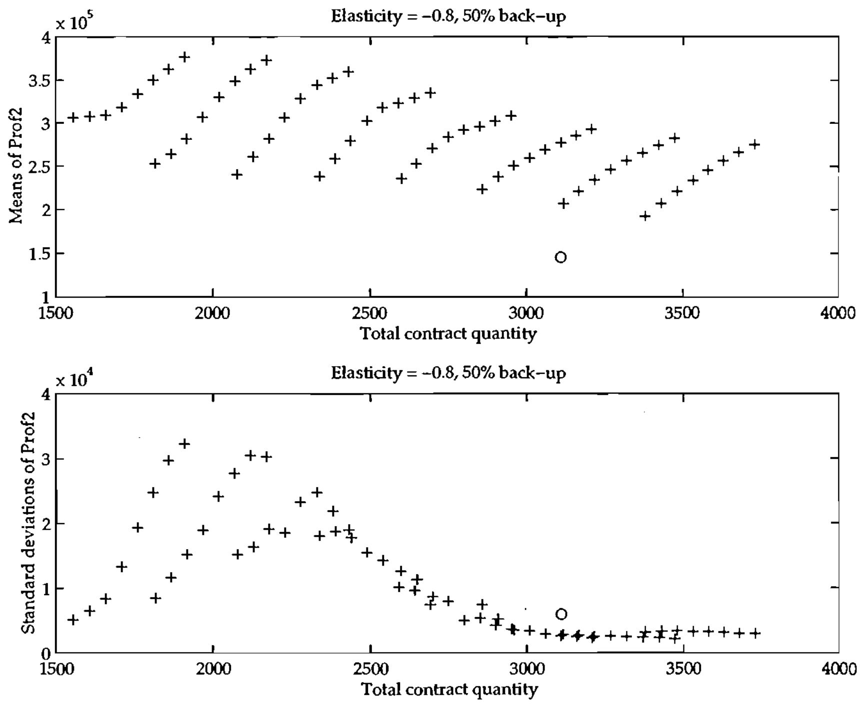

Figure A.54: Means and standard deviations of profit, Firm Two. Elasticity = -0.8, 50% back-up.

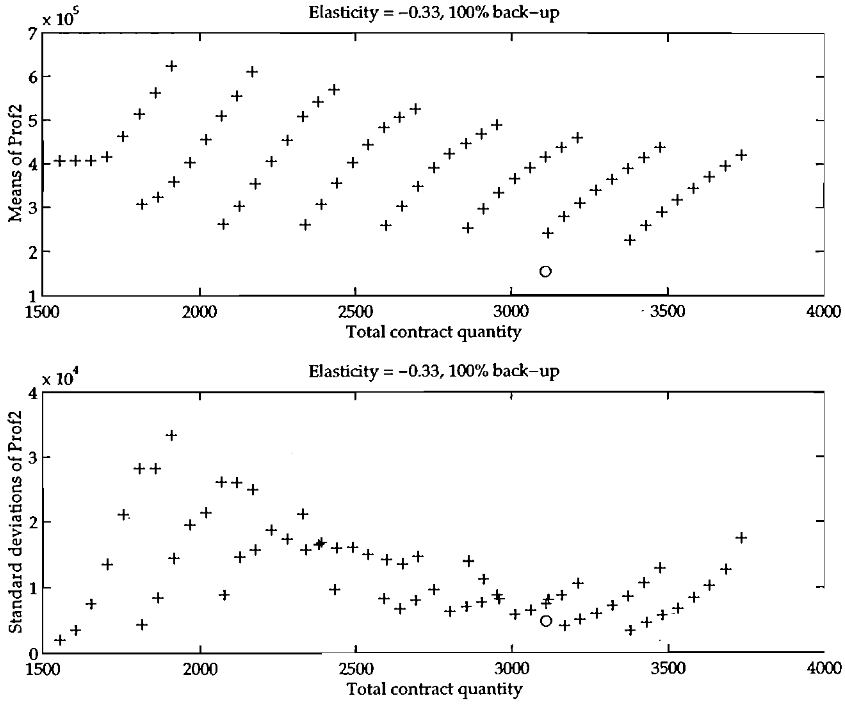

Figure A.55: Means and standard deviations of profit, Firm Two. Elasticity = -0.33, 100% back-up.

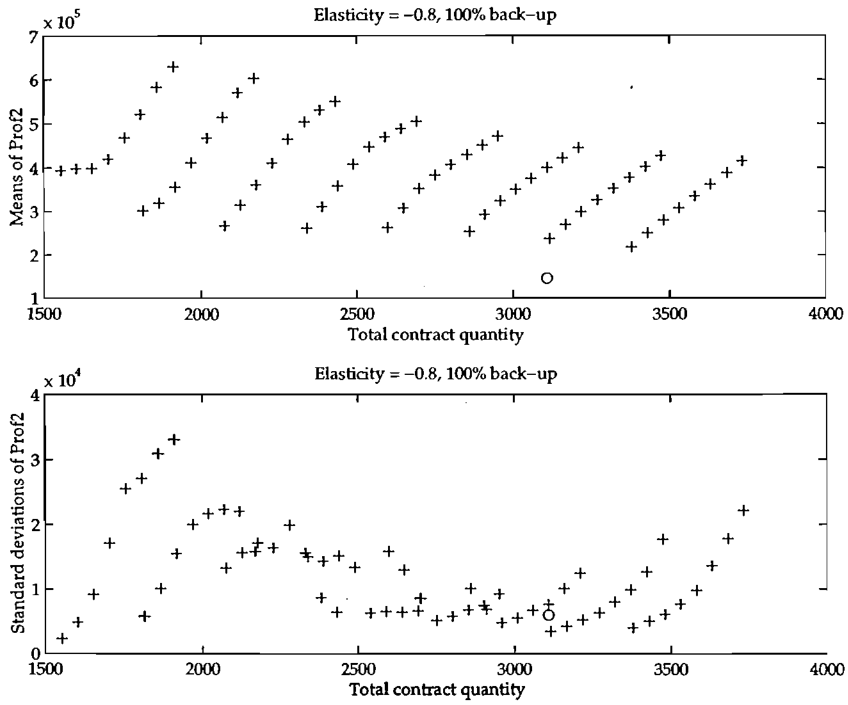

Figure A.56: Means and standard deviations of profit, Firm Two. Elasticity = -0.8, 100% back-up.

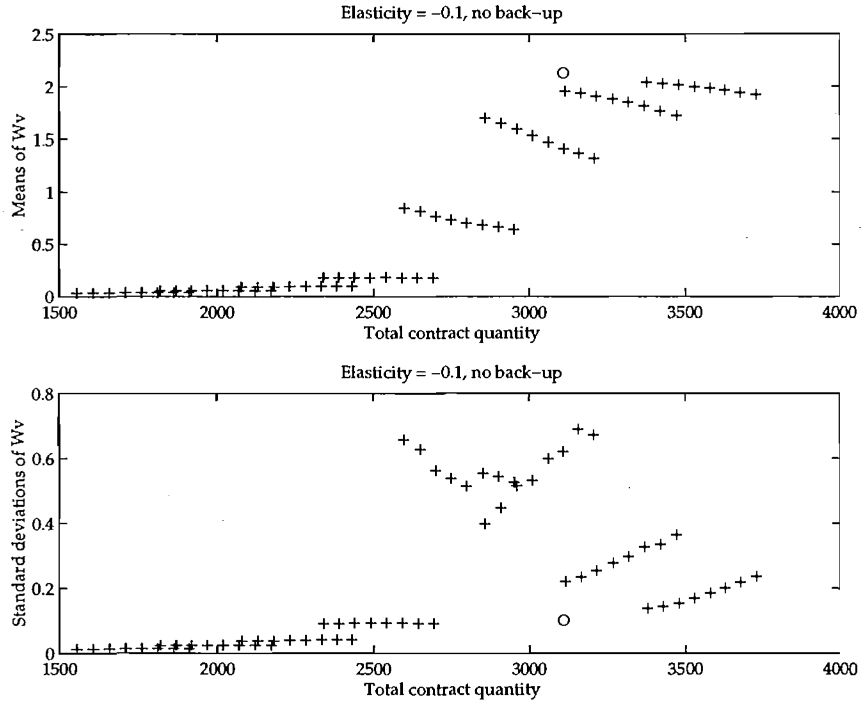

Figure A.57: Means and standard deviations of marginal water value. Elasticity = -0.1, no back-up.

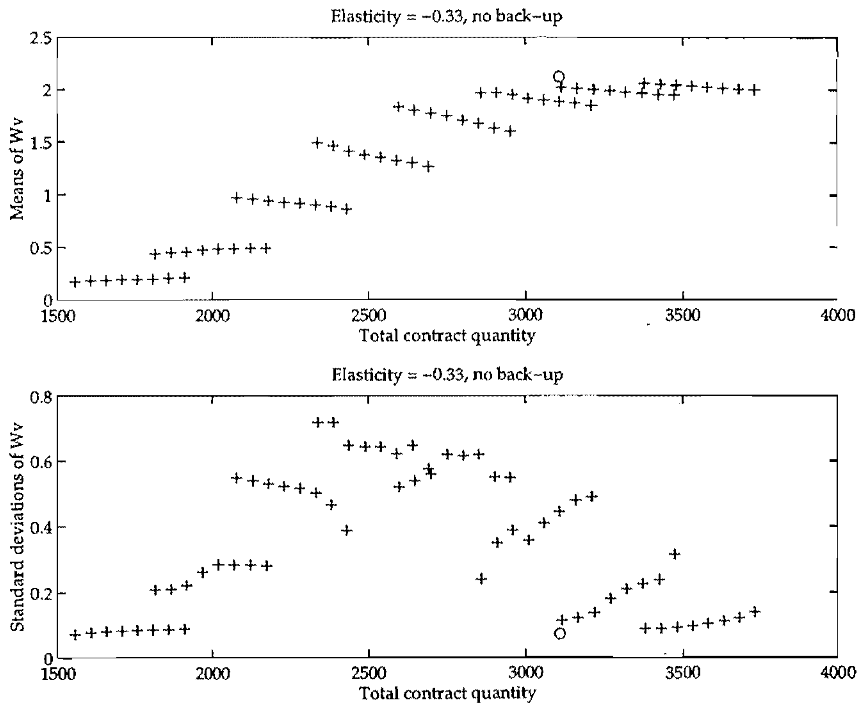

Figure A.58: Means and standard deviations of marginal water value. Elasticity = -0.33, no back-up.

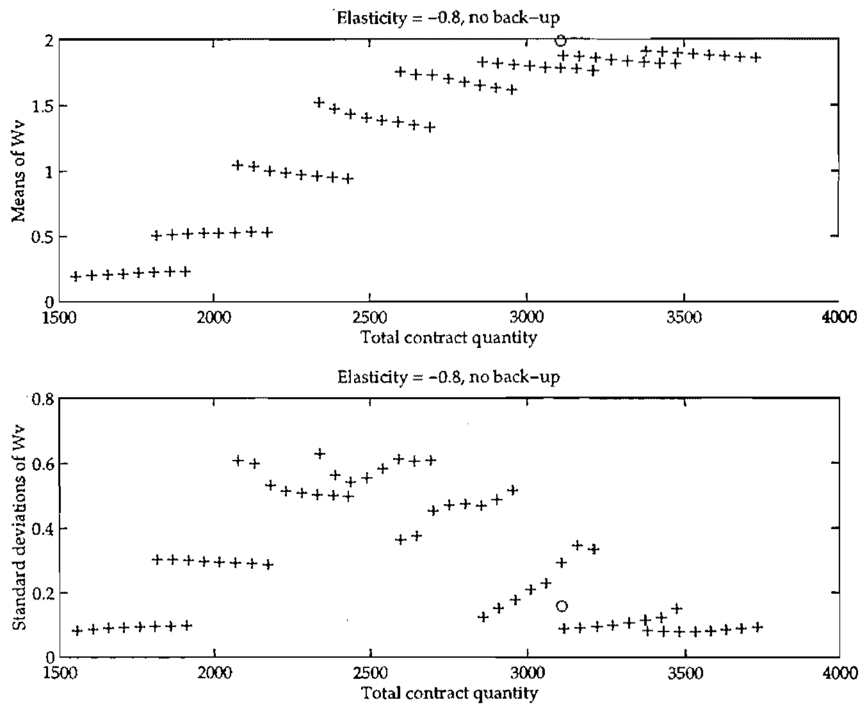

Figure A.59: Means and standard deviations of marginal water value. Elasticity = -0.8, no back-up.

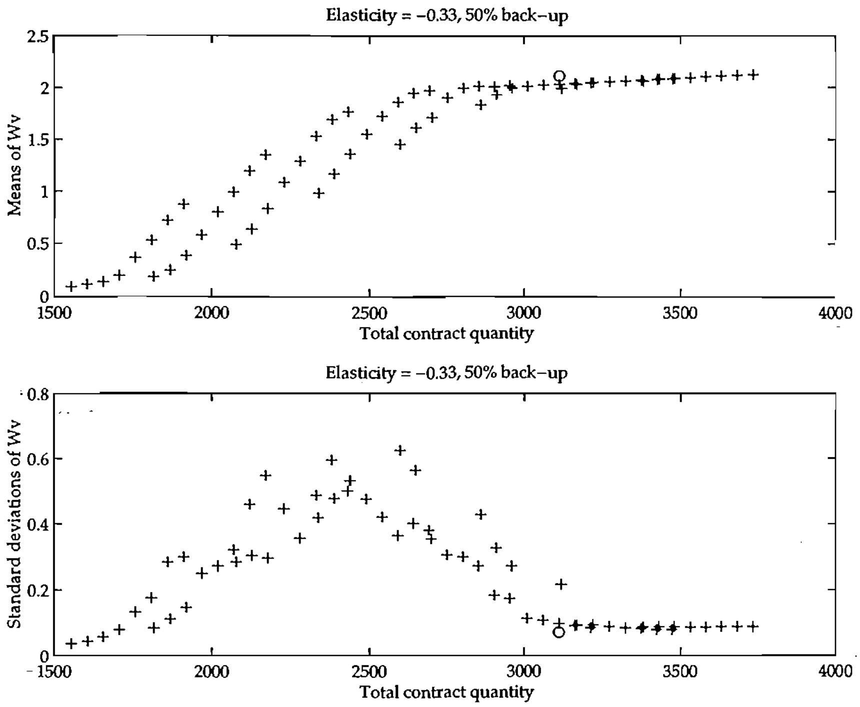

Figure A.60: Means and standard deviations of marginal water value. Elasticity = -0.33, 50% back-up.

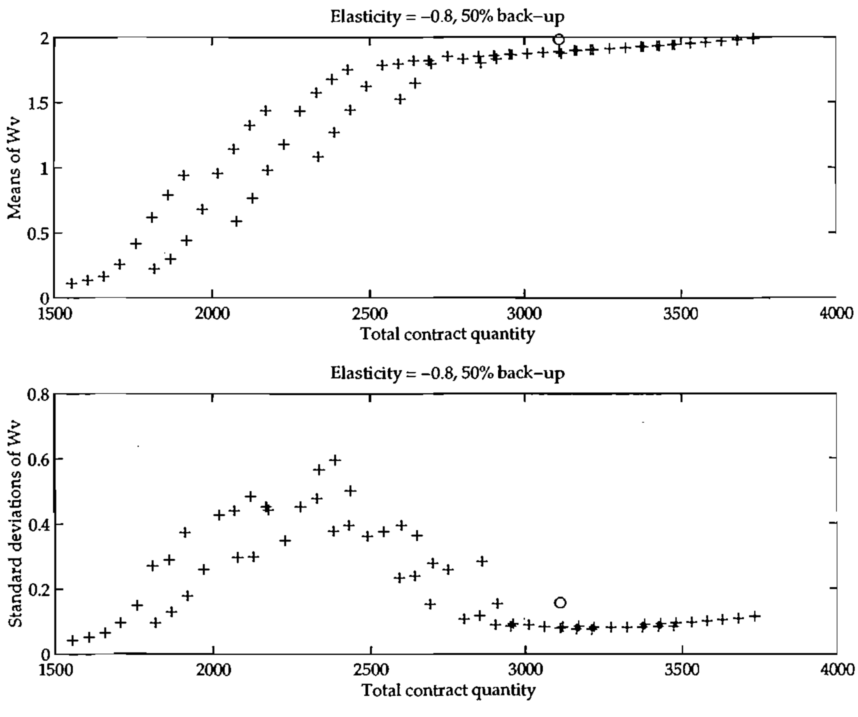

Figure A.61: Means and standard deviations of marginal water value. Elasticity = -0.8, 50% back-up.

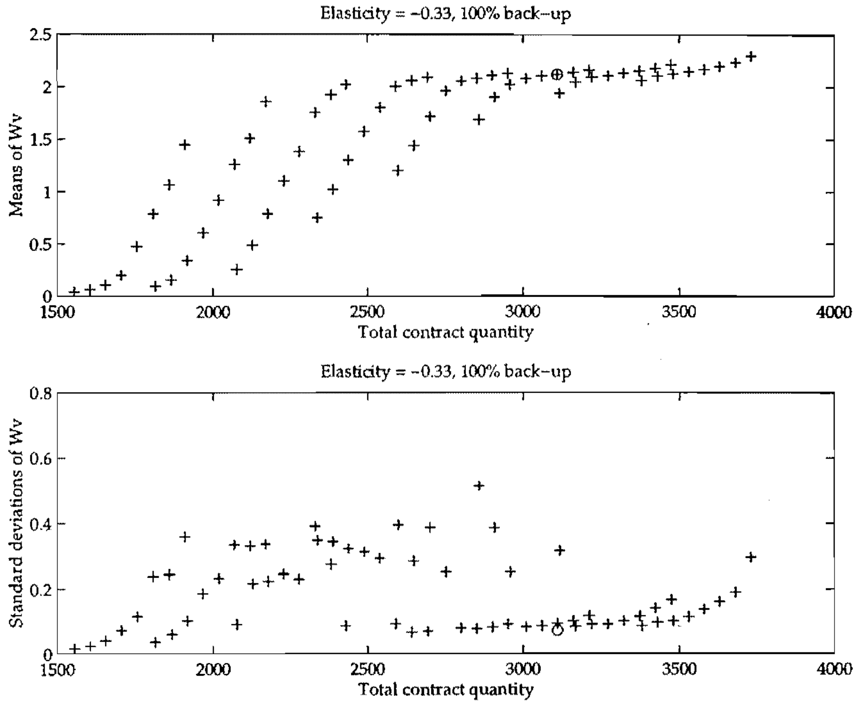

Figure A.62: Means and standard deviations of marginal water value. Elasticity = -0.33, 100% back-up.

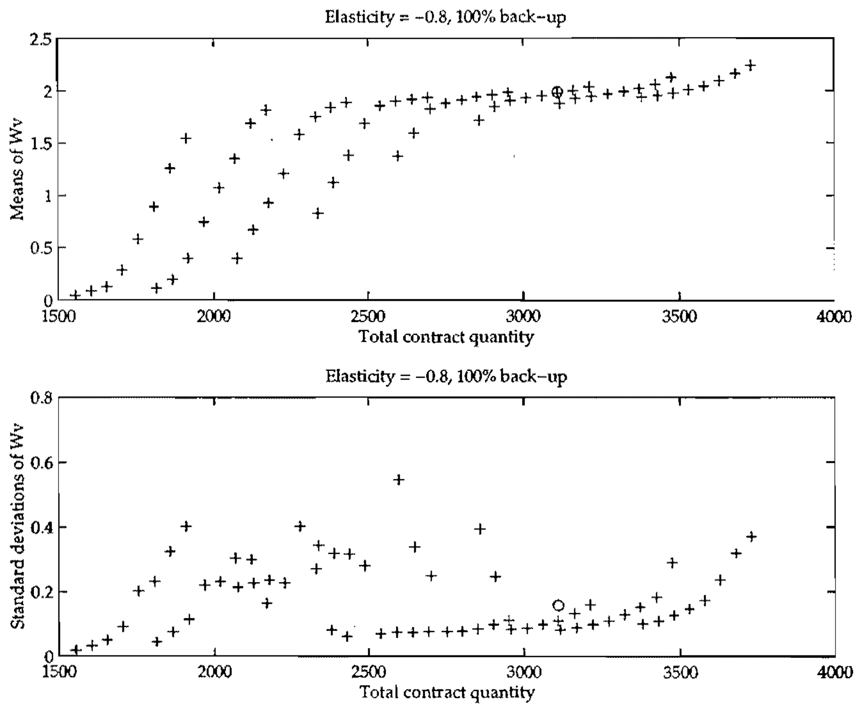

Figure A.63: Means and standard deviations of marginal water value. Elasticity = -0.8, 100% back-up.

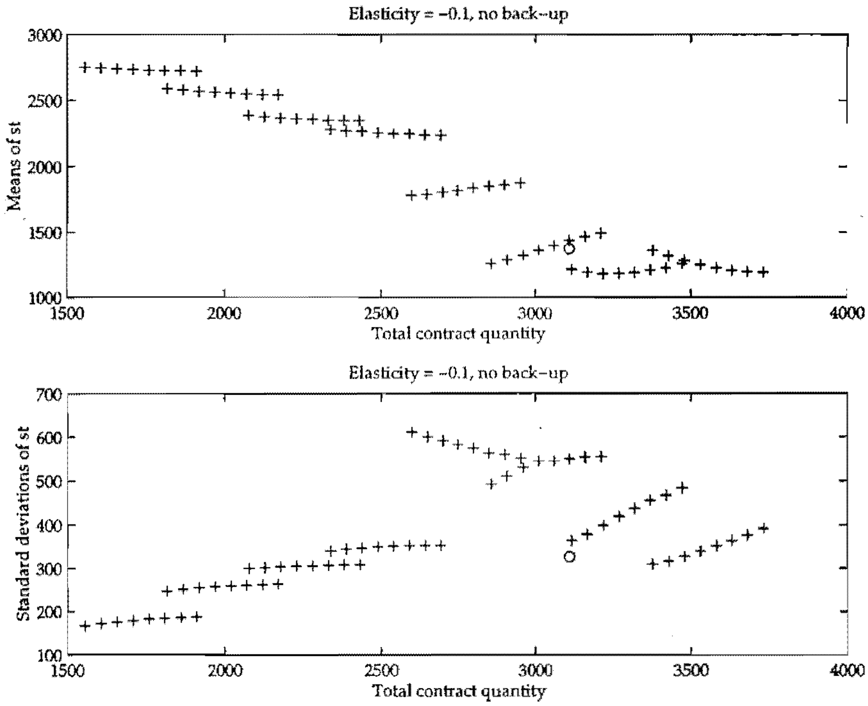

Figure A.64: Means and standard deviations of storage. Elasticity = -0.1, no back-up.

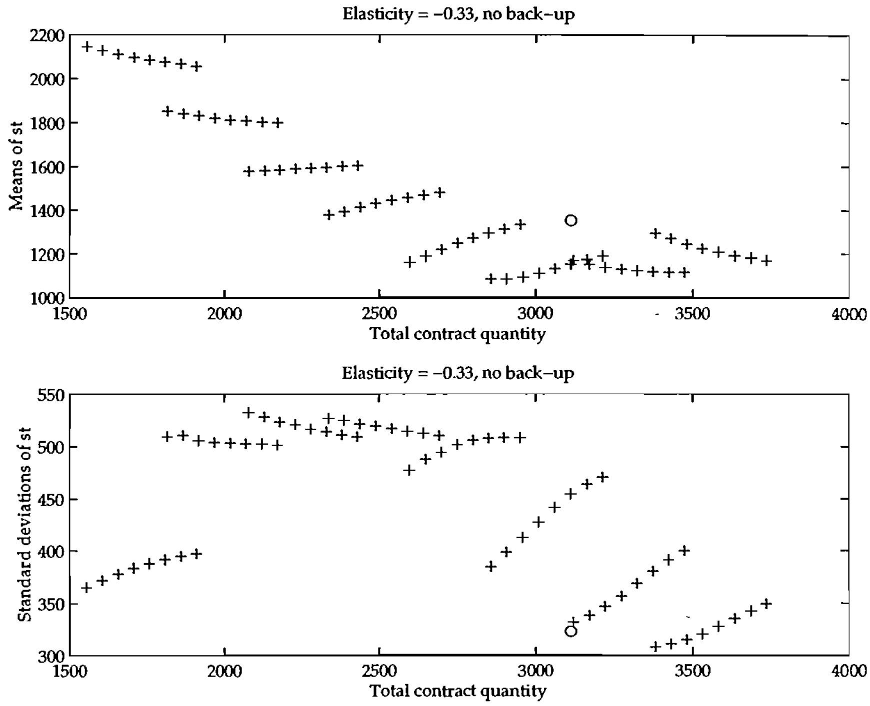

Figure A.65: Means and standard deviations of storage. Elasticity = -0.33, no back-up.

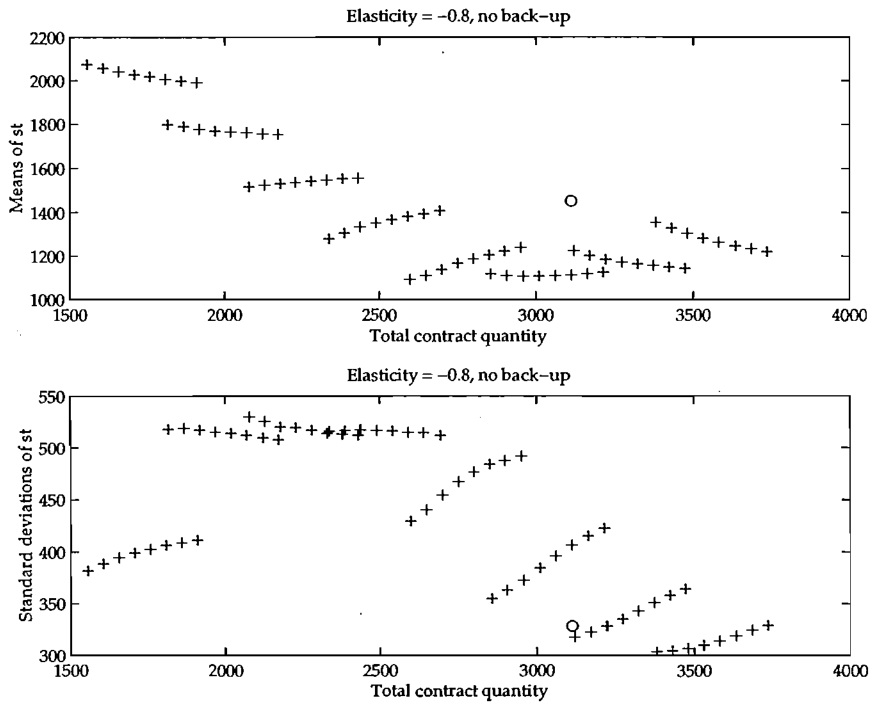

Figure A.66: Means and standard deviations of storage. Elasticity = -0.8, no back-up.

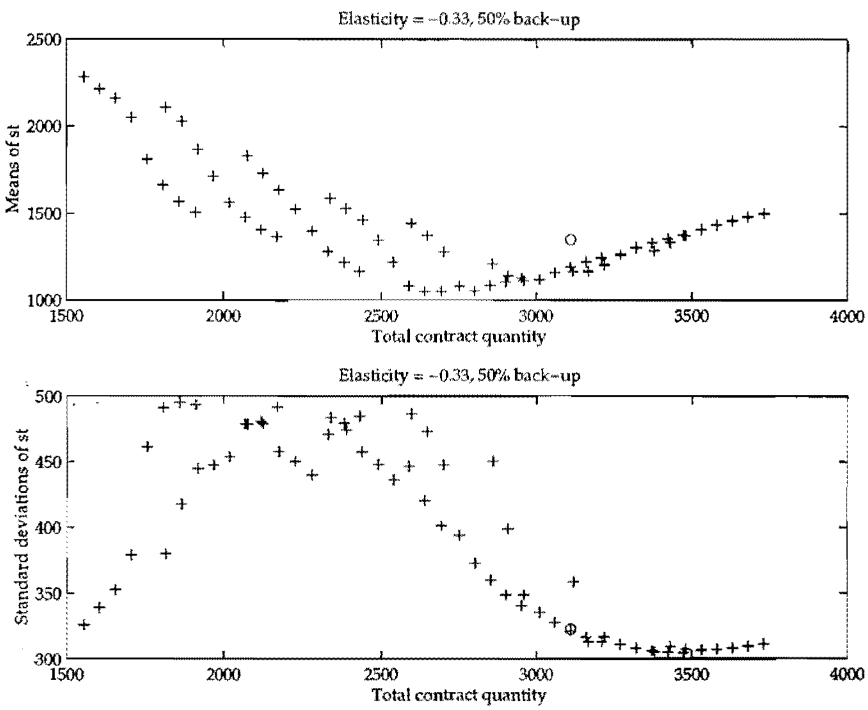

Figure A.67: Means and standard deviations of storage. Elasticity = -0.33, 50% back-up.

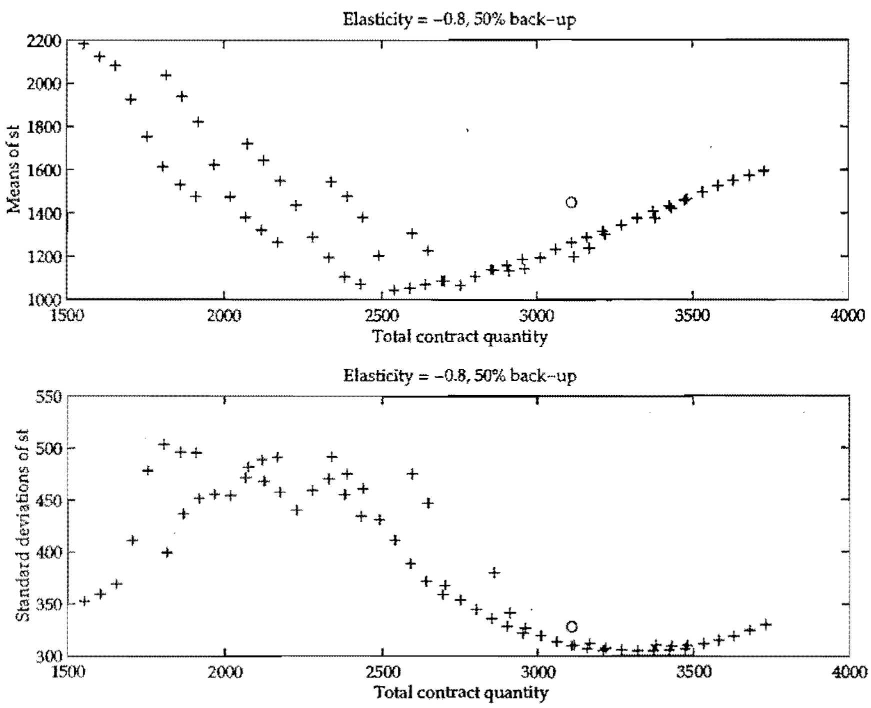

Figure A.68: Means and standard deviations of storage. Elasticity = -0.8, 50% back-up.

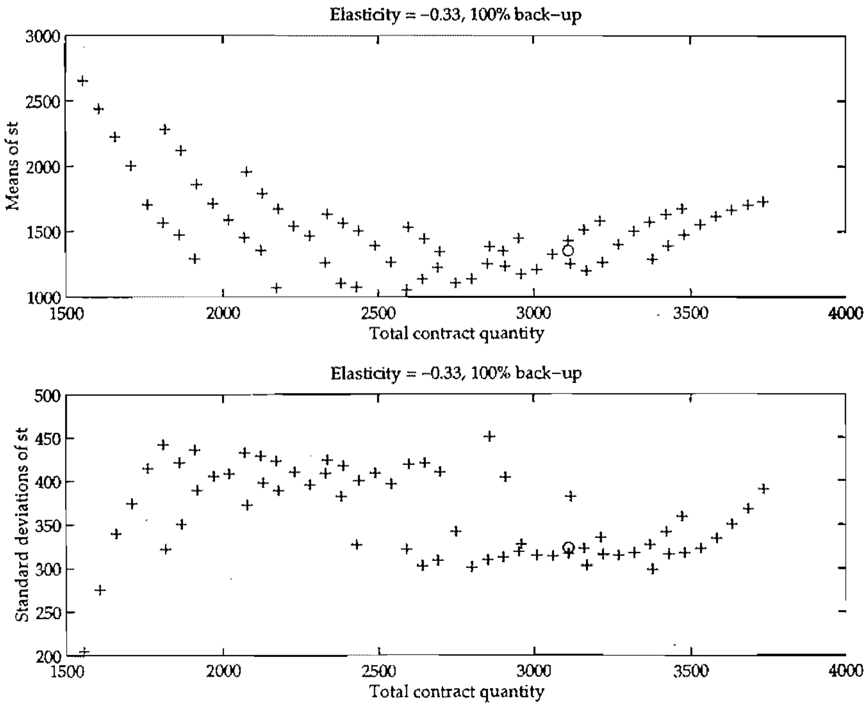

Figure A.69: Means and standard deviations of storage. Elasticity = -0.33, 100% back-up.

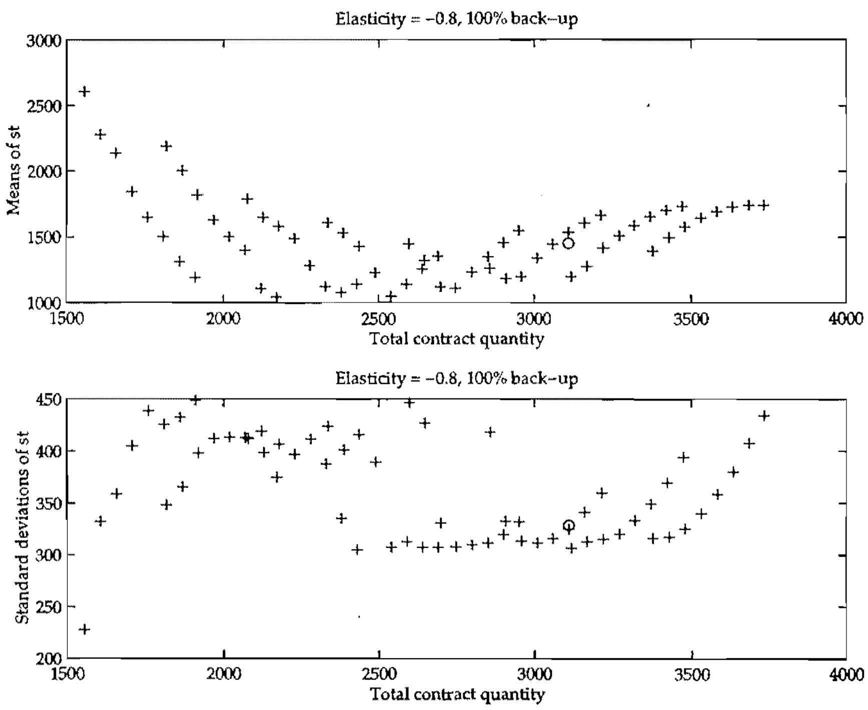

Figure A.70: Means and standard deviations of storage. Elasticity = -0.8, 100% back-up.

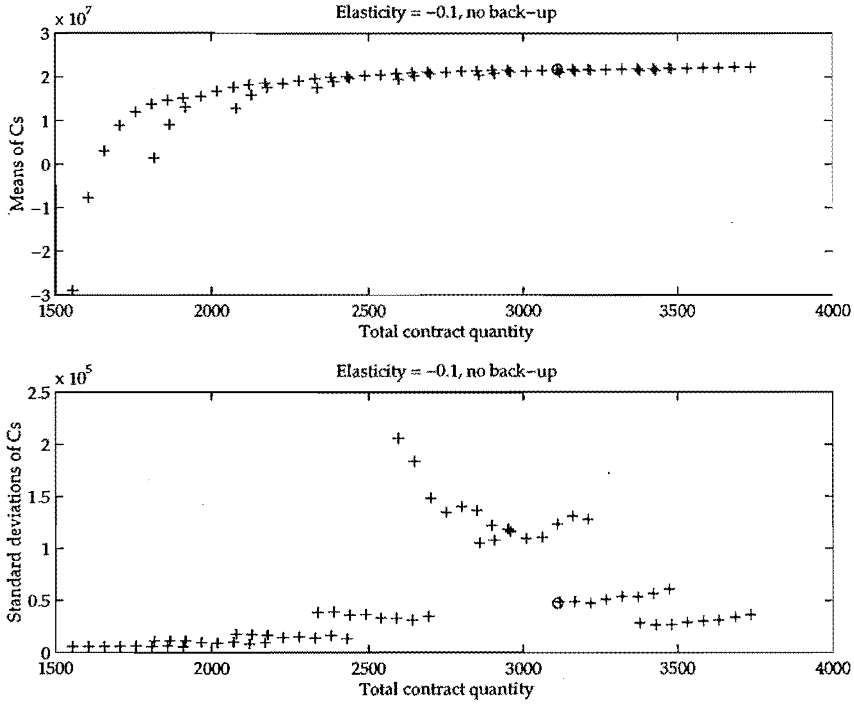

Figure A.71: Means and standard deviations of Consumer Surplus. Elasticity = -0.1, no back-up.

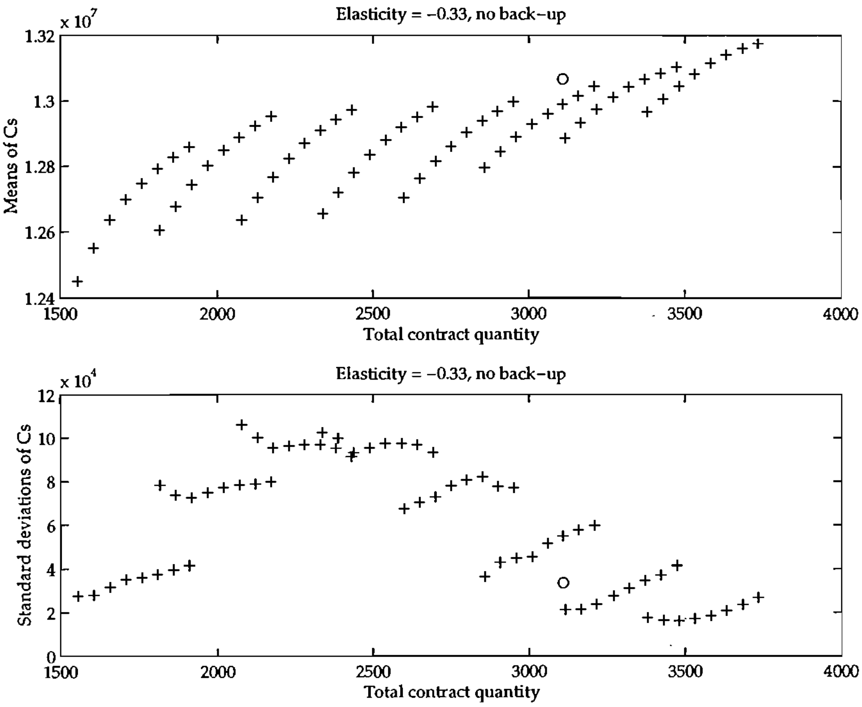

Figure A.72: Means and standard deviations of Consumer Surplus. Elasticity = -0.33, no back-up.

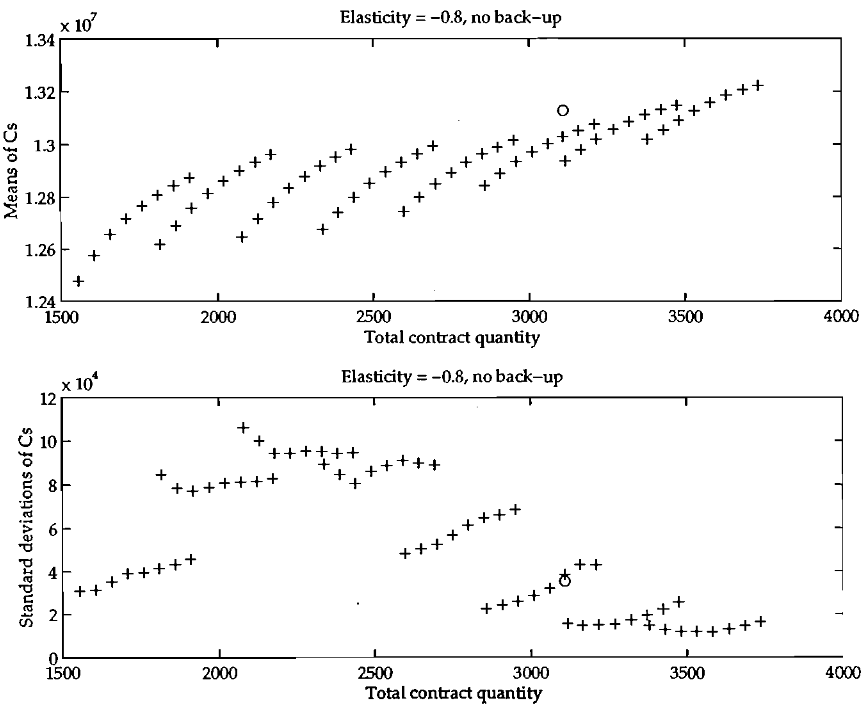

Figure A.73: Means and standard deviations of Consumer Surplus. Elasticity = -0.8, no back-up.

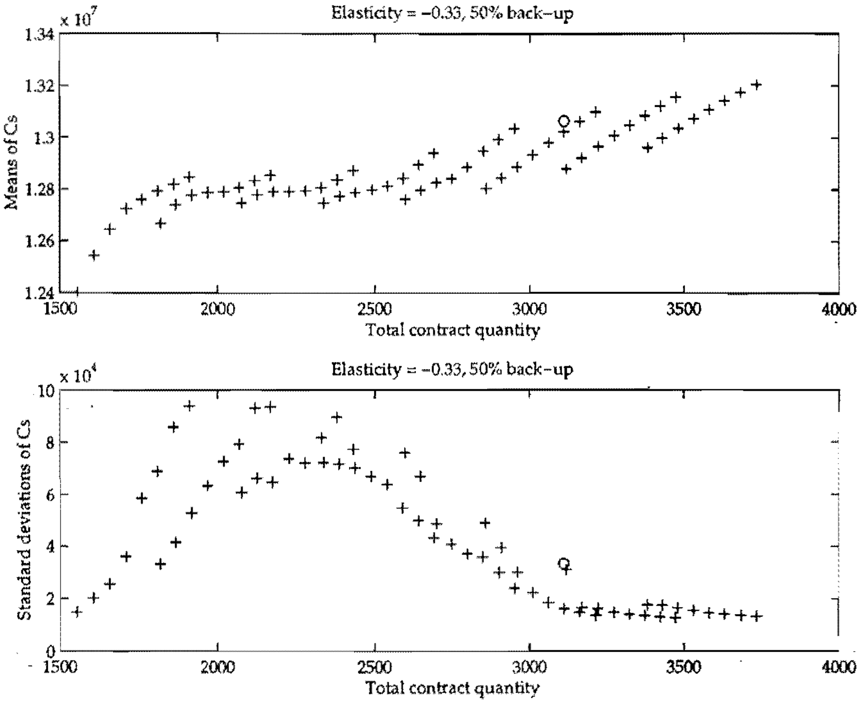

Figure A.74: Means and standard deviations of Consumer Surplus. Elasticity = -0.33, 50% back-up.

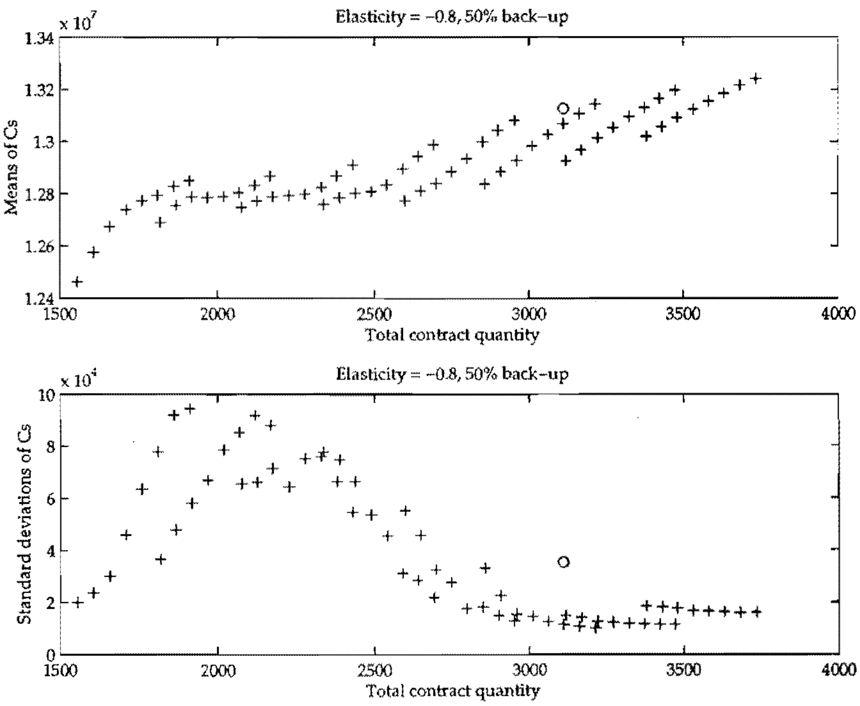

Figure A.75: Means and standard deviations of Consumer Surplus. Elasticity = -0.8, 50% back-up.

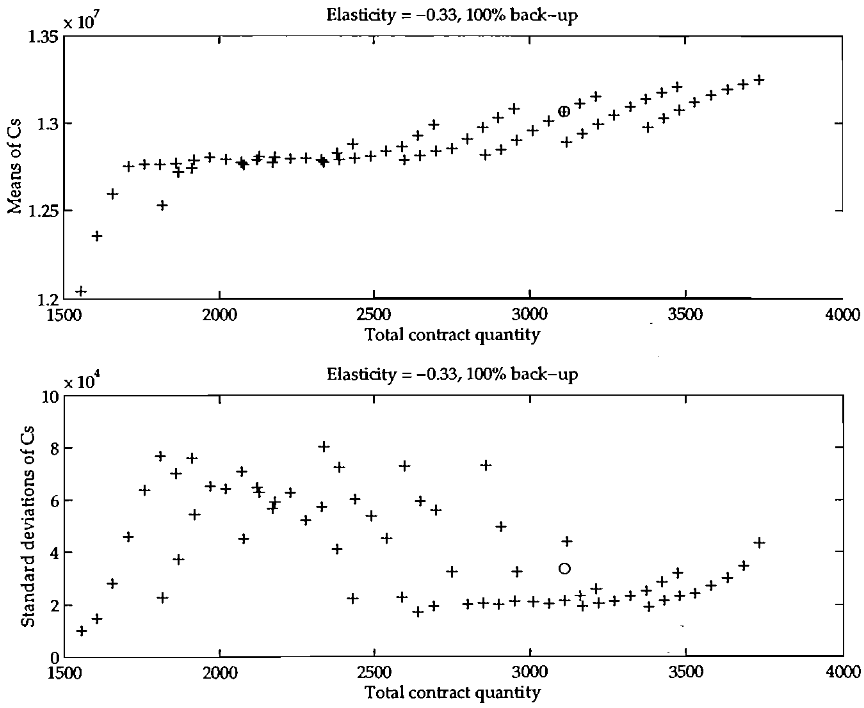

Figure A.76: Means and standard deviations of Consumer Surplus. Elasticity = -0.33, 100% back-up.

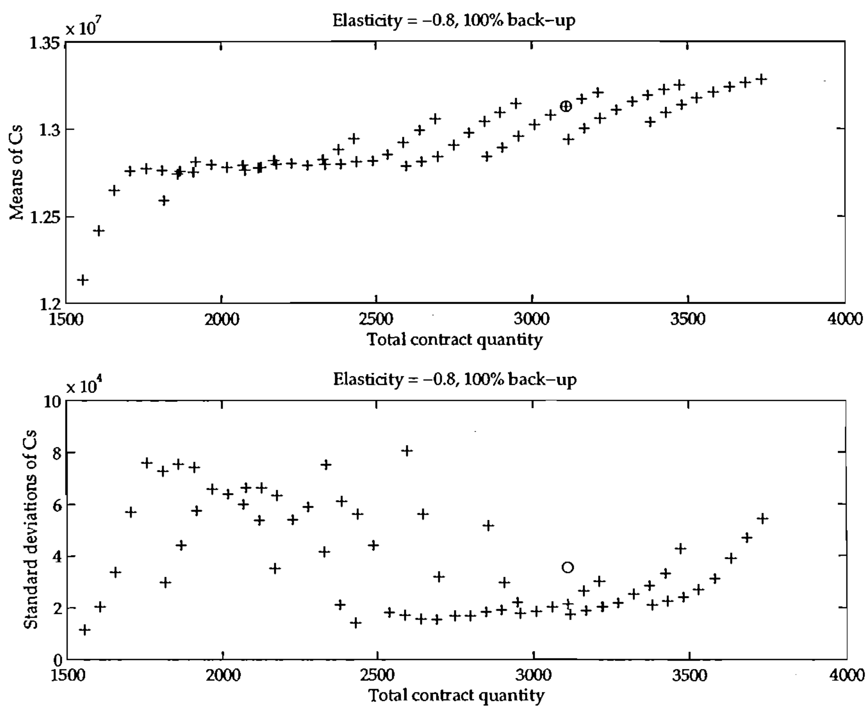

Figure A.77: Means and standard deviations of Consumer Surplus. Elasticity = -0.8, 100% back-up.

A.2 Results for the *ww* model

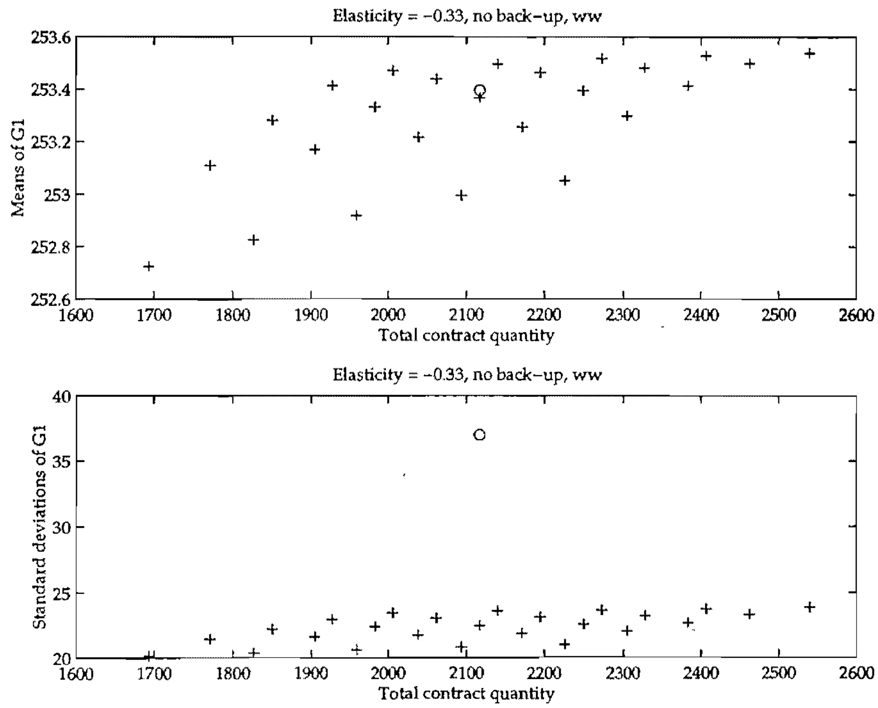

Figure A.78: Means and standard deviations of total generation, Firm One. Elasticity = -0.33, no back-up.

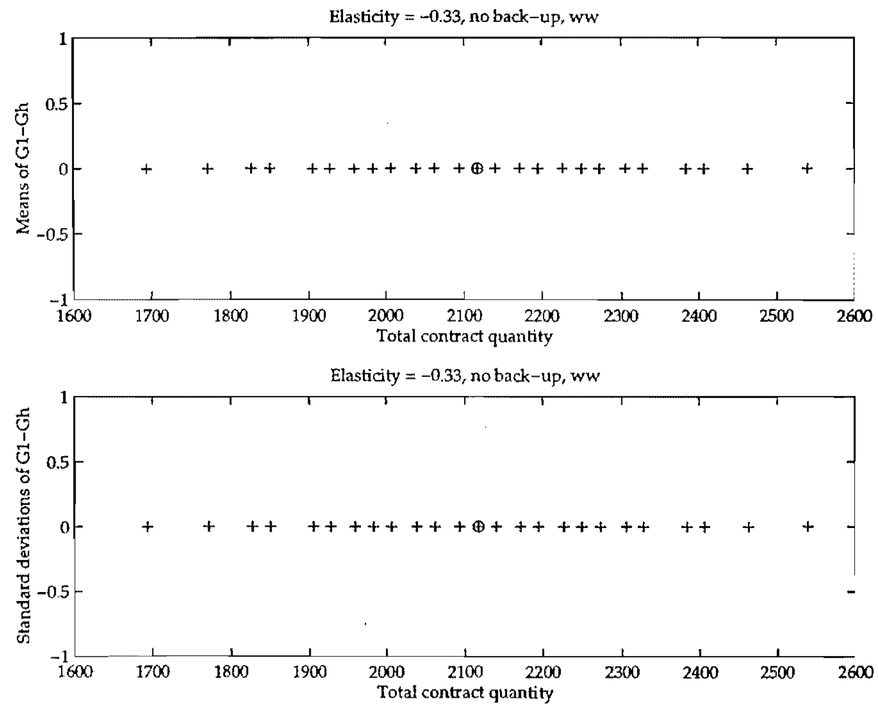

Figure A.79: Means and standard deviations of thermal generation, Firm One. Elasticity = -0.33, no back-up.

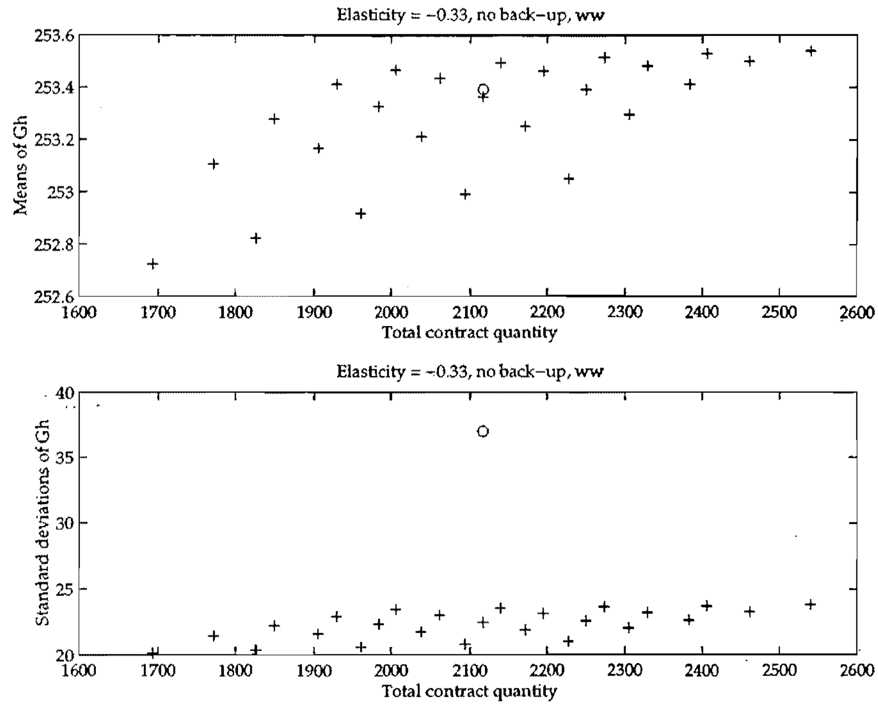

Figure A.80: Means and standard deviations of hydro generation. Elasticity = -0.33, no back-up.

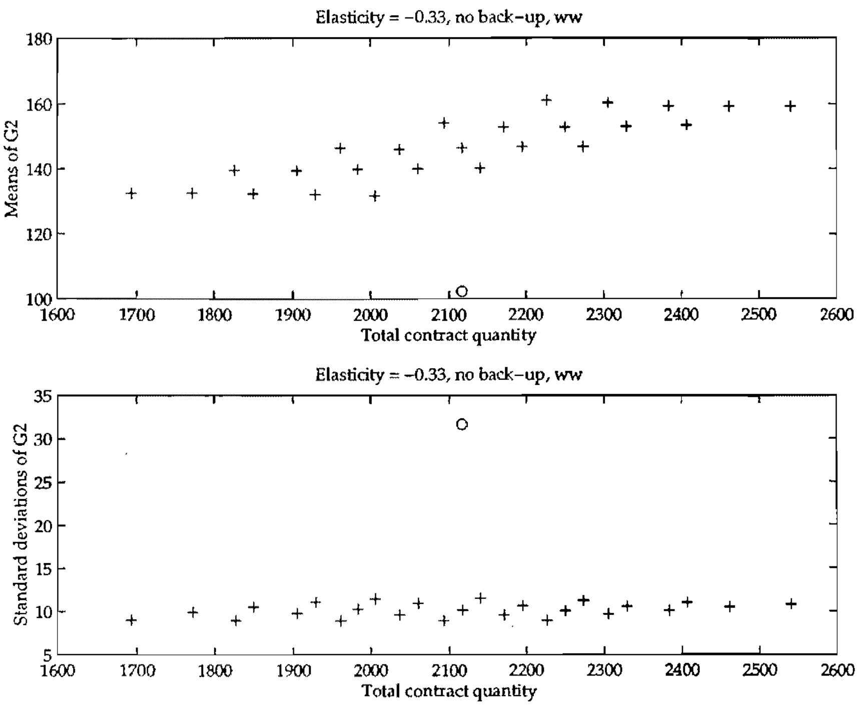

Figure A.81: Means and standard deviations of generation, Firm Two. Elasticity = -0.33, no back-up.

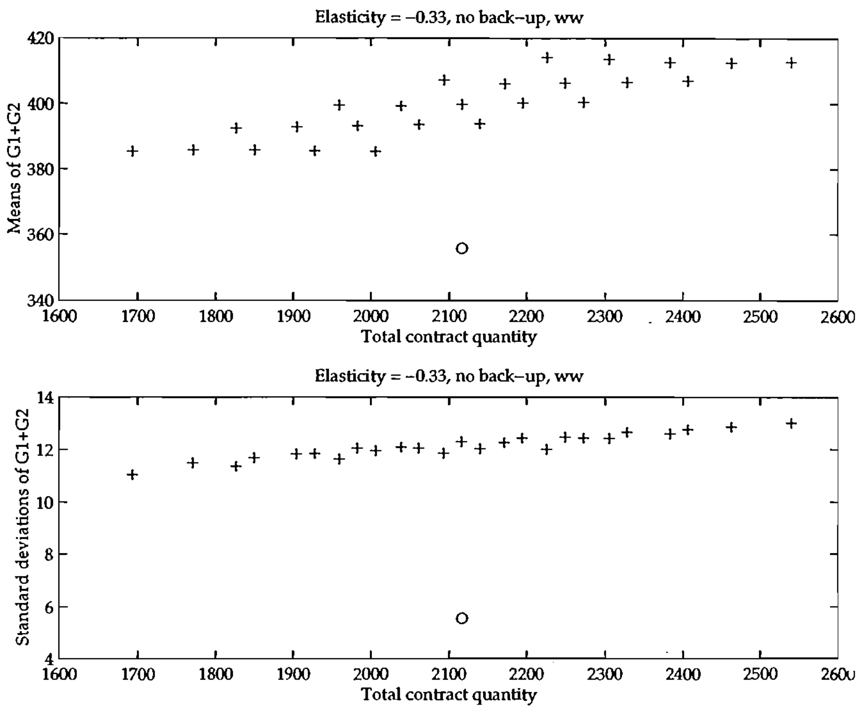

Figure A.82: Means and standard deviations of total generation, Firm One plus Firm Two. Elasticity = -0.33, no back-up.

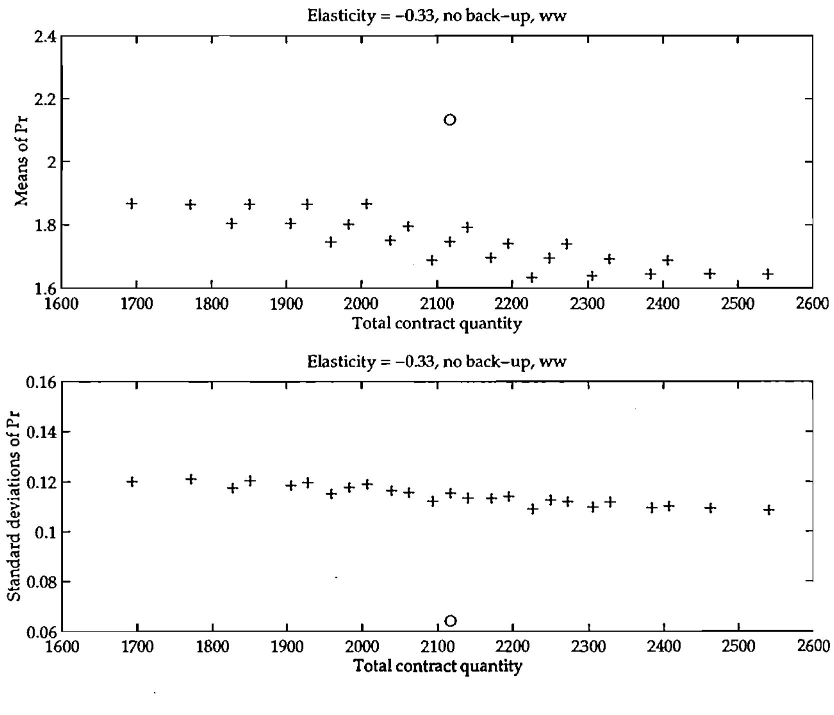

Figure A.83: Means and standard deviations of energy spot price. Elasticity = -0.33, no back-up.

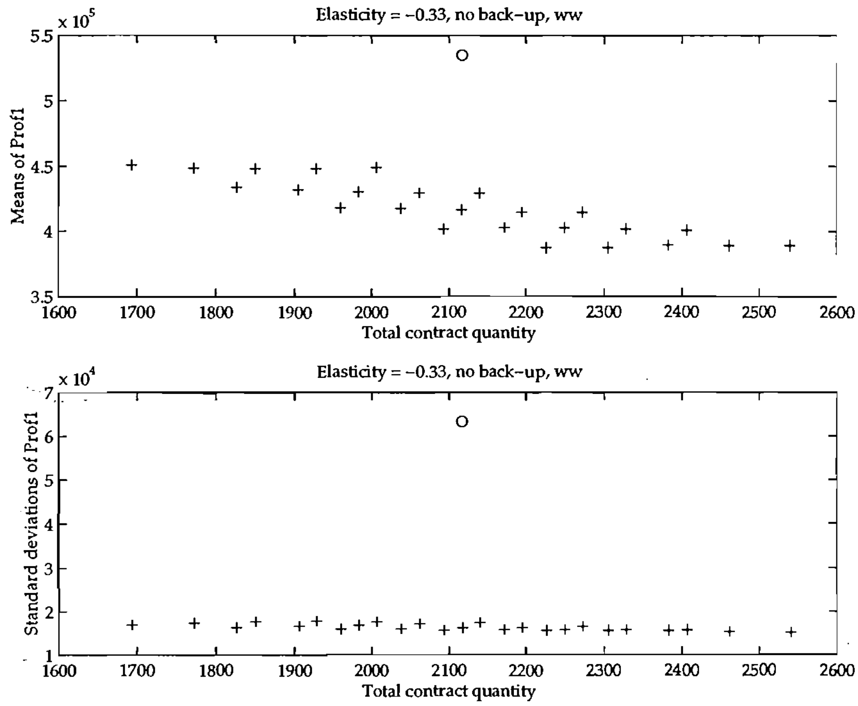

Figure A.84: Means and standard deviations of profit, Firm One. Elasticity = -0.33, no back-up.

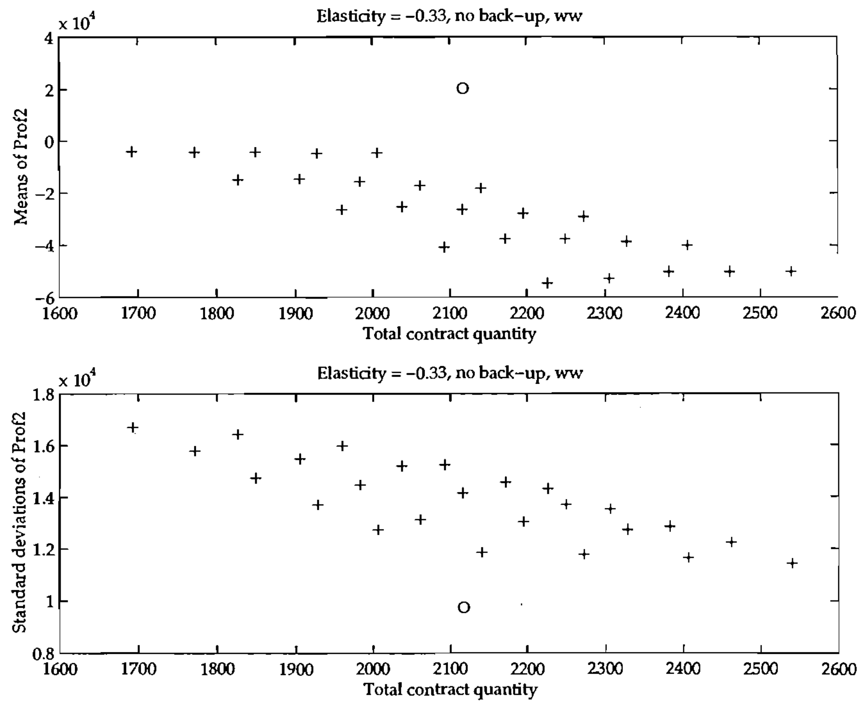

Figure A.85: Means and standard deviations of profit, Firm Two. Elasticity = -0.33, no back-up.

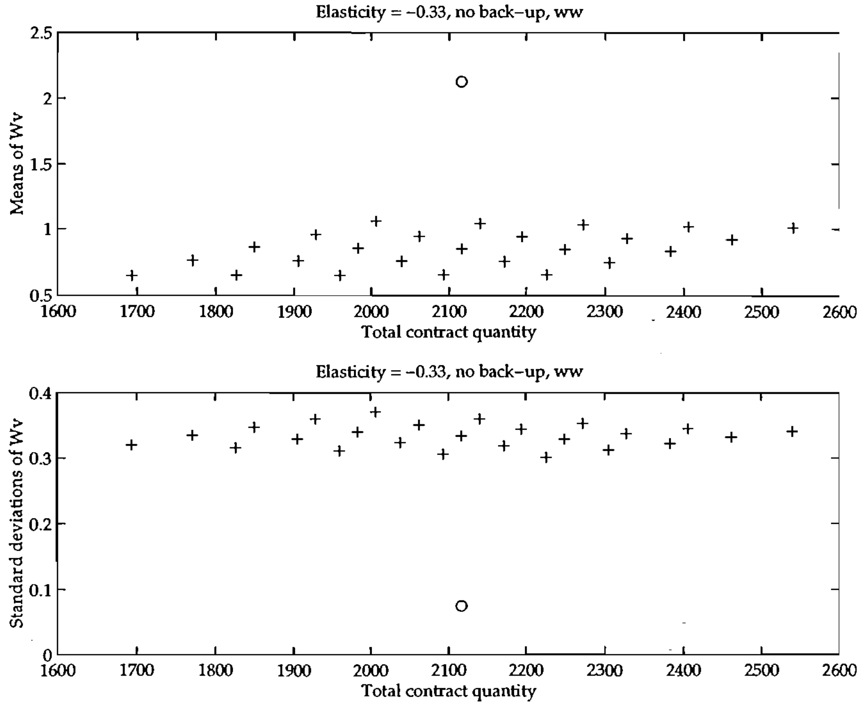

Figure A.86: Means and standard deviations of marginal water value. Elasticity = -0.33, no back-up.

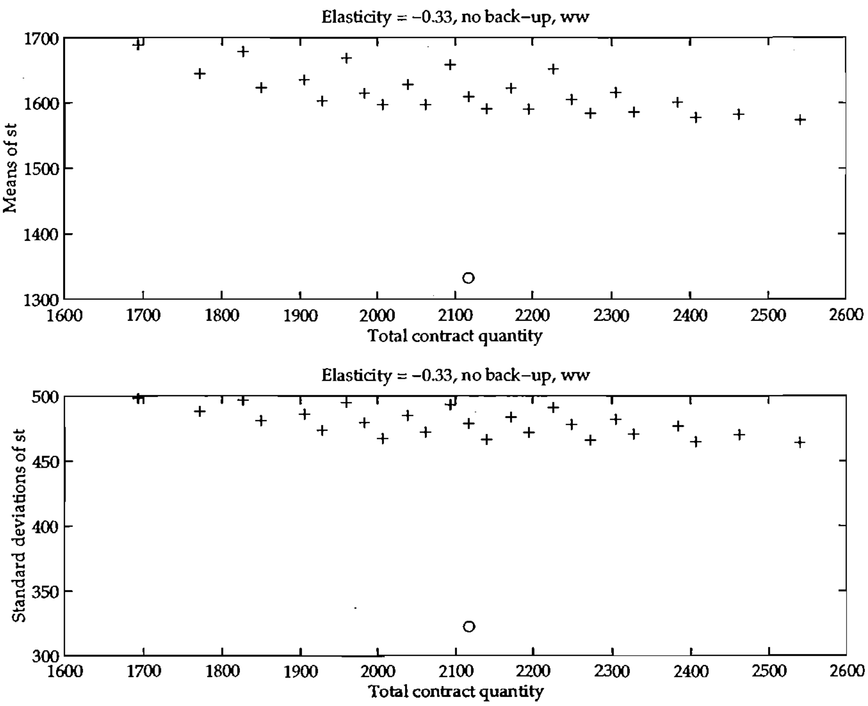

Figure A.87: Means and standard deviations of storage. Elasticity = -0.33, no back-up.

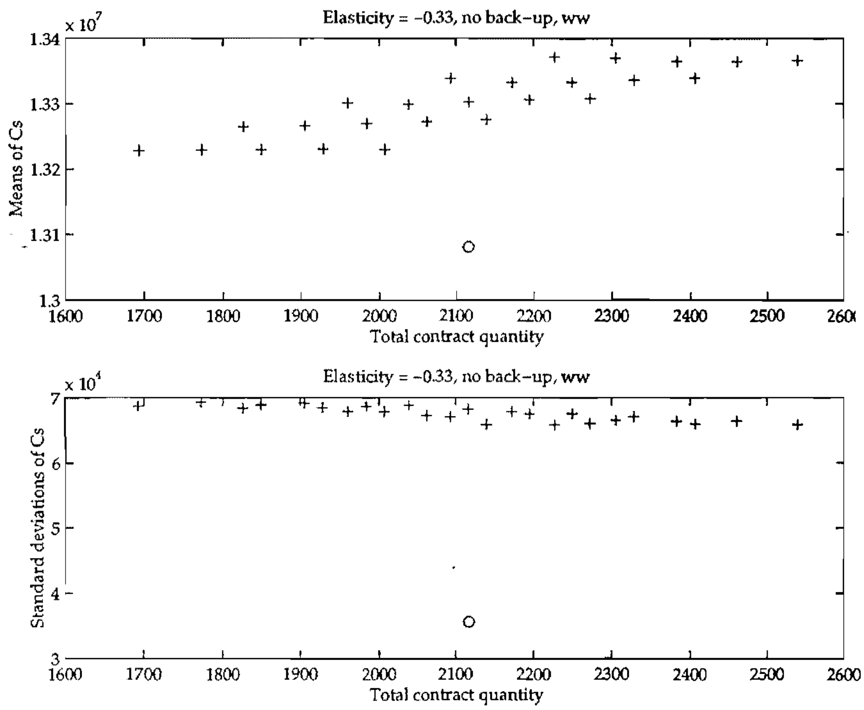

Figure A.88: Means and standard deviations of Consumer Surplus. Elasticity = -0.33, no back-up.